1. **(2 points)** Give the equations for finding the center of mass over a region \( R \) with density function \( \delta(x, y) \) in rectangular coordinates.

**Solution:**

\[
\bar{x} = \frac{\int \int_R x \delta(x, y) \, dA}{\int \int_R \delta(x, y) \, dA}
\]

\[
\bar{y} = \frac{\int \int_R y \delta(x, y) \, dA}{\int \int_R \delta(x, y) \, dA}
\]

2. **(8 points)** Find the centroid of the region in the polar coordinates plane defined by the inequalities \( 0 \leq r \leq 3 \) and \( -\frac{\pi}{3} \leq \theta \leq \frac{\pi}{3} \).

**Solution:** To find the centroid of \( R \), we set \( \delta = 1 \) and find the center of mass.

\( R \) is symmetric with respect to the \( x \)-axis (if you aren’t convinced of this, draw a picture) and so \( \bar{y} = 0 \).

To find \( \bar{x} \) we use the equation we found in Problem 1. \( R \) is described in polar coordinates by the inequalities \( 0 \leq r \leq 3 \) and \( -\frac{\pi}{3} \leq \theta \leq \frac{\pi}{3} \). Also, \( dA \) is described in polar coordinates by \( r \, dr \, d\theta \). Therefore the denominator is given by

\[
\int \int_R \delta(x, y) \, dA = \int_{\theta=-\pi/3}^{\pi/3} \int_{r=0}^{3} r \, dr \, d\theta = \int_{\theta=-\pi/3}^{\pi/3} \left[ \frac{1}{2} r^2 \right]_{r=0}^{3} d\theta
\]

\[
= \int_{\theta=-\pi/3}^{\pi/3} \frac{9}{2} d\theta = \left[ \frac{9}{2} \theta \right]_{\theta=-\pi/3}^{\pi/3} = 3\pi
\]

Also, using the fact that \( x = r \cos \theta \), the numerator is given by

\[
\int \int_R x \delta(x, y) \, dA = \int_{\theta=-\pi/3}^{\pi/3} \int_{r=0}^{3} r^2 \cos \theta \, dr \, d\theta = \int_{\theta=-\pi/3}^{\pi/3} \left[ \frac{1}{3} r^3 \cos \theta \right]_{r=0}^{3} d\theta
\]

\[
= \int_{\theta=-\pi/3}^{\pi/3} 9 \cos \theta \, d\theta = 9 \left[ \sin \theta \right]_{\theta=-\pi/3}^{\pi/3}
\]

\[
= 9 \left[ \frac{\sqrt{3}}{2} - \left( -\frac{\sqrt{3}}{2} \right) \right] = 9\sqrt{3}
\]

Putting this all together, we have that the \( x \) coordinate of the centroid is given by

\[
\bar{x} = \frac{\int \int_R x \delta(x, y) \, dA}{\int \int_R \delta(x, y) \, dA} = \frac{9\sqrt{3}}{3\pi} = \frac{3\sqrt{3}}{\pi}
\]

Therefore, the centroid of \( R \) is \((\bar{x}, \bar{y}) = \left( \frac{3\sqrt{3}}{\pi}, 0 \right)\).