Let \( f(x, y) = x^2y + \sqrt{x} \sin(xy) \)

**Hint:** You may use the fact that the partial derivatives \( \frac{\partial^2 f}{\partial y \partial x} \) and \( \frac{\partial^2 f}{\partial x \partial y} \) are continuous at \((x, y) = (2, \frac{\pi}{8})\).

a. (7 points) Evaluate the partial derivative \( \frac{\partial^2 f}{\partial x \partial y} \) at the point \((x, y) = (2, \frac{\pi}{8})\).

**Solution:** To find \( \frac{\partial^2 f}{\partial x \partial y} \) we must first calculate \( \frac{\partial f}{\partial y} \).

\[
\frac{\partial f}{\partial y} = \frac{\partial}{\partial y} \left[ x^2y + \sqrt{x} \sin(xy) \right] = x^2 + \frac{x^{3/2}}{2} \cos(xy)
\]

We now calculate \( \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x} \left( \frac{\partial f}{\partial y} \right) \).

\[
\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x} \left[ x^2 + \frac{x^{3/2}}{2} \cos(xy) \right] = 2x + \frac{3}{2}x^{1/2} \cos(xy) - \frac{x^{3/2}}{2} y \sin(xy)
\]

Finally, we evaluate at the point \((x, y) = (2, \frac{\pi}{8})\).

\[
\frac{\partial^2 f}{\partial x \partial y} \bigg|_{(2,\pi/8)} = 2(2) + \frac{3}{2} \sqrt{2} \cos \left( \frac{\pi}{4} \right) - (2)^{3/2} \left( \frac{\pi}{8} \right) \sin \left( \frac{\pi}{4} \right)
\]

\[
= 4 + \frac{3}{2} - \frac{\pi}{4} = \frac{11}{2} - \frac{\pi}{4} = \frac{22 - \pi}{4}
\]

b. (3 points) Evaluate the partial derivative \( \frac{\partial^2 f}{\partial y \partial x} \) at the point \((x, y) = (2, \frac{\pi}{8})\).

**Solution:**

One method is to find \( \frac{\partial f}{\partial x} \) and then directly calculate \( \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial}{\partial y} \left( \frac{\partial f}{\partial x} \right) \).

However, following the hint yields the solution much more quickly. Since the derivatives \( \frac{\partial^2 f}{\partial y \partial x} \) and \( \frac{\partial^2 f}{\partial x \partial y} \) are continuous at \((x, y) = (2, \frac{\pi}{8})\), we can conclude that they are equal at \((x, y) = (2, \frac{\pi}{8})\). Therefore

\[
\frac{\partial^2 f}{\partial y \partial x} \bigg|_{(2,\pi/8)} = \frac{\partial^2 f}{\partial x \partial y} \bigg|_{(2,\pi/8)} = \frac{11}{2} - \frac{\pi}{4} = \frac{22 - \pi}{4}
\]