Consider the integral \[ \int_0^2 \int_{\sqrt{y/2}}^1 6xy\sqrt{x^6 + 1} \, dx \, dy. \]

a. (4 points) Rewrite the integral with the order of integration reversed.

**Solution:** From the sketch, we see that \( x \) runs from 0 to 1 and that at each fixed value of \( x \), \( y \) runs from 0 to \( 2x^2 \). Therefore our integral will become

\[
\int_{x=0}^{x=1} \int_{y=0}^{y=2x^2} 6xy\sqrt{x^6 + 1} \, dy \, dx
\]

b. (6 points) Use your answer from part (a) to evaluate the integral.

**Solution:**

\[
\int_{x=0}^{x=1} \int_{y=0}^{y=2x^2} 6xy\sqrt{x^6 + 1} \, dy \, dx = \int_{x=0}^{x=1} \left[ 3xy^2\sqrt{x^6 + 1} \right]_{y=0}^{y=2x^2} \, dx
\]

\[
= \int_{x=0}^{x=1} 12x^5\sqrt{x^6 + 1} \, dx
\]

We then solve this using the substitution \( u = x^6 + 1 \). Then we have that \( du = 6x^5 \, dx \). Also, when \( x = 0 \) we have \( u = 1 \) and when \( x = 1 \) we have \( u = 2 \). Therefore we are left with

\[
\int_{x=0}^{x=1} \int_{y=0}^{y=2x^2} 6xy\sqrt{x^6 + 1} \, dy \, dx = \int_{u=1}^{u=2} 2u^{1/2} \, du
\]

\[
= 2 \left[ \frac{2}{3} u^{3/2} \right]_{u=1}^{u=2} = \frac{4}{3} [ (2)^{3/2} - 1 ]
\]