1. (1 points) If $z = a + bi$ what do we call $a - bi$?

Solution: It is known as the complex conjugate of $z$ and can be written $\bar{z}$.

2. (4 points) Evaluate the following expressions. Your final answer should be in the form $a + bi$. a. $(2 + i)(\sqrt{3} - 4i)$

Solution: We distribute the expression, keeping in mind that $i^2 = -1$.
So $(2 + i)(\sqrt{3} - 4i) = 2\sqrt{3} + \sqrt{3}i - 8i + 4 = (2\sqrt{3} + 4) + (\sqrt{3} - 8)i$

b. $\frac{4}{1 - 3i}$

Solution: We recall that to divide two complex numbers, we multiply the top and bottom by the complex conjugate of the denominator. So

$$\frac{4}{1 - 3i} = \frac{4}{1 - 3i} \left(\frac{1 + 3i}{1 + 3i}\right) = \frac{4 + 12i}{1 + 9} = \frac{2}{5} + \frac{6}{5}i$$

3. (4 points) Evaluate $\left(\frac{1}{2} + \frac{1}{2}i\right)^{10}$ using polar coordinates.

Solution: We note that, in polar coordinates we have $r = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = \frac{1}{\sqrt{2}}$. Also, the angle that $\left(\frac{1}{2} + \frac{1}{2}i\right)$ makes with the positive $x$-axis is $\theta = \frac{\pi}{4}$ (draw a picture if you aren’t convinced of this). So we have that

$$\left(\frac{1}{2} + \frac{1}{2}i\right)^{10} = \left(\frac{1}{\sqrt{2}}\right)^{10} \left(\cos\left(10\left(\frac{\pi}{4}\right)\right) + \sin\left(10\left(\frac{\pi}{4}\right)\right)i\right)$$

$$= \frac{1}{32} \left(\cos\left(\frac{5\pi}{2}\right) + \sin\left(\frac{5\pi}{2}\right)i\right)$$

$$= \frac{1}{32}i$$

4. (1 points) Simplify $e^{i\pi} + 1$.

Solution: We have that $e^{i\pi} + 1 = \cos(\pi) + \sin(\pi)i + 1 = -1 + 1 = 0$. 