Let \( f(x, y) = x \ln y + \frac{e^{xy^2}}{y} \)

**Hint:** You may use the fact that the partial derivatives \( \frac{\partial^2 f}{\partial y \partial x} \) and \( \frac{\partial^2 f}{\partial x \partial y} \) are continuous at \((x, y) = (\ln 2, 1)\).

**a. (7 points)** Evaluate the partial derivative \( \frac{\partial^2 f}{\partial y \partial x} \) at the point \((x, y) = (\ln 2, 1)\).

**Solution:** To find \( \frac{\partial^2 f}{\partial y \partial x} \) we must first calculate \( \frac{\partial f}{\partial x} \).

\[
\frac{\partial f}{\partial x} = \frac{\partial}{\partial x} \left[ x \ln y + \frac{e^{xy^2}}{y} \right] = \ln y + \frac{y^2 e^{xy^2}}{y} = \ln y + ye^{xy^2}
\]

We now calculate \( \frac{\partial^2 f}{\partial y \partial x} = \frac{\partial}{\partial y} \left( \frac{\partial f}{\partial x} \right) \).

\[
\frac{\partial^2 f}{\partial y \partial x} = \frac{\partial}{\partial y} \left[ \ln y + ye^{xy^2} \right] = \frac{1}{y} + e^{xy^2} + y \left( 2ye^{xy^2} \right) = \frac{1}{y} + e^{xy^2} + 2y^2 e^{xy^2}
\]

Finally, we evaluate at the point \((x, y) = (\ln 2, 1)\).

\[
\left. \frac{\partial^2 f}{\partial y \partial x} \right|_{(\ln 2, 1)} = 1 + 2(\ln 2)e^{\ln^2} + e^{\ln^2} = 3 + 4 \ln 2 = 3 + \ln 8
\]

**b. (3 points)** Evaluate the partial derivative \( \frac{\partial^2 f}{\partial x \partial y} \) at the point \((x, y) = (\ln 2, 1)\).

**Solution:**

One method is to find \( \frac{\partial f}{\partial y} \) and then directly calculate \( \frac{\partial^2 f}{\partial x \partial y} = \frac{\partial}{\partial x} \left( \frac{\partial f}{\partial y} \right) \).

However, following the hint yields the solution much more quickly. Since the derivatives \( \frac{\partial^2 f}{\partial y \partial x} \) and \( \frac{\partial^2 f}{\partial x \partial y} \) are continuous at \((x, y) = (\ln 2, 1)\), we can conclude that they are equal at \((x, y) = (\ln 2, 1)\). Therefore

\[
\left. \frac{\partial^2 f}{\partial x \partial y} \right|_{(\ln 2, 1)} = \left. \frac{\partial^2 f}{\partial y \partial x} \right|_{(\ln 2, 1)} = 3 + 4 \ln 2 = 3 + \ln 8
\]