

Instructions: Complete all problems. You must provide explanations for your work in order to receive credit. Circle your answers. If a question is unclear, please do not hesitate to ask me for a clarification.

Name_____

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Problem 2	
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Total	

1. Ron and Hermione are filling up the water bowls for their pets, Ron's pet mouse Scabbers and Hermione's pet cat Crookshanks. Both bowls hold 80 ounces of water, and while Crookshanks' bowl is empty, Scabbers' bowl has 38 ounces of water already in it. They both cast the "Aguamenti" spell, which causes water to come out of their wands. As Hermione is the better caster of the two, water comes out of her wand at the rate of 10 ounces per minute, while Ron's spell causes the water to come out at a rate of 6 ounces per minute.

- (a) (10 pts.) Write formulas for the amount of water in each of the bowls (one formula for each bowl). Explain how you got your formula.

Answer: The formula for Crookshanks' bowl is $y = 10x$, where y represents the amount of water, x represents the time in minutes, and 10 comes from the speed of Hermione's Aguamenti spell. The formula for Scabbers' bowl is $y = 6x + 38$. y and x mean the same thing, 6 comes from the speed of Ron's Aguamenti spell, and 38 is the amount of water already in the bowl.

- (b) (15 pts.) Will Crookshanks' bowl ever have more water than Scabbers' bowl? If so, when? If not, explain why not.

Answer: No, it will not. One way to see this is to set the equations equal, to see when they have the same amount. You get $10x = 6x + 38$, or $4x = 38$, or $x = 9.5$. Thus, the time when they would have the same amount of water is 9.5 minutes. If you plug this back into one of the equations, you get that the amount of water at this time would be 95 ounces. But this is more than either bowl could hold. Thus, Crookshanks' bowl will never have more water.

- (c) (20 pts.) When will Scabbers' bowl fill up? Crookshanks' bowl? Explain how you got your answer.

Answer: By setting $y = 80$ in both equations and solving for x , we find that Scabbers' bowl fills up at 7 minutes, and Crookshanks' bowl fills up in 8 minutes.

2. You work at a shop which makes rods of various lengths. These rods are made by gluing cubes together (see the picture below). Your job is to put stickers on each visible face that appears. We will denote a_n as the number of stickers needed for a rod made up of n many cubes (e.g. a_1 will be the number of stickers needed for a rod made up of one cube, a_2 will be the number needed for a rod of two cubes, etc.).

- (a) (5 pts.) What is a_1 ? a_2 ? a_3 ? a_4 ?

Answer: $a_1 = 6, a_2 = 10, a_3 = 14, a_4 = 18$.

- (b) (10 pts.) Give the recursive formula for a_n . Explain how you got the formula.

Answer: The recursive formula is $a_1 = 6$, and $a_{n+1} = a_n + 4$.

- (c) (20 pts.) Write two different closed formulas for a_n . Explain how these formulas are present in the picture above.

Answer: People came up with many answers. The two most popular were $a_n = 4n + 2$ and $a_n = 6n - 2(n - 1)$. The first comes from the idea that each block has 4 faces visible, the front, top, back, and bottom. The two ends are unaccounted for, so we get $a_n = 4n + 2$. The second formula comes from the idea that each block has 6 faces. However, the blocks touch each other $n - 1$ times, and each of these touches removes 2 faces. Thus, we get $6n - 2(n - 1)$.

3. A cookie recipe calls for 3 teaspoons of salt and 1 cup of sugar.

- (a) (15 pts.) If you have $2\frac{1}{2}$ teaspoons of salt and want to make as many cookies as you can, how much sugar will you need? Solve this two different ways. If you set two quantities equal, be sure to justify that step.

Answer: There are many ways to answer this. Almost everyone used the fact that $\frac{3}{1} = \frac{2.5}{x}$. We can use this because the quantities of sugar and salt are proportional, so the amount of salt divided by the amount of sugar must be constant. Solving this, we get $x = \frac{5}{6}$.

Another way to solve this is to say $\frac{3}{2.5} = \frac{1}{x}$. We can do this, because the factor that we multiply 3 by to get 2.5, we must multiply 1 by to get x . Again, if we solve this, we get $x = \frac{5}{6}$.

- (b) (15 pts.) Suppose you have π^2 teaspoons of salt. How many cups of sugar do you need? Solve this in two different ways.

Answer: Using the same technique as above, we get that we need $\frac{\pi^2}{3}$ cups of sugar.

4. (a) Suppose x and y are quantities that are proportionally related.

i. (10 pts.) What does the equation relating x and y look like? How does this show that the two quantities are proportional?

Answer: The equation is $y = mx$. This clearly shows the quantities are proportional since $\frac{y}{x} = m$.

ii. (10 pts.) What does a graph of this relationship look like? What special properties does this graph have?

Answer: This graph is a straight line, but has the special property that it goes through the origin.

(b) Suppose x and y are quantities that are linearly related, but not proportionally related.

i. (10 pts.) What does the equation relating x and y look like? How does this show that the two quantities are linearly related?

Answer: The equation is $y = mx + b$. This shows the quantities are linearly related because $\frac{\Delta y}{\Delta x} = m$.

ii. (10 pts.) What does a graph of this relationship look like?

Answer: A straight line, but does not go through the origin.