Math 132
Practice problems 2 Ben Ellison

Remember that the **expected value** of an experiment is defined as the sum of the probability of an outcome times the numerical value of an outcome. Of course, only experiments with numerical outcomes can have an expected value, but every experiment with numerical outcomes will have an expected value.

Here are some sample problems:

1. Find the expected value of the product of rolling two dice.

2. The Wisconsin Pick 3 Lotto works as follows. You pick three numbers from 0 to 9 in a particular order. Note that you can have repeats: $1-2-3$ is a valid choice, as is $4-4-9$. It costs \$1 to play, and the pay out is \$500. Figure out the expected value of playing the Pick 3.

3. Suppose you play the following game: there are two boxes, one of which has twice as much money as the other (but you don't know the exact money amounts). You pick a box and open it, and find \$100. Thus, the other box either contains \$50 or \$200. You are offered the choice of switching boxes, but if you switch, you can't go back to your box. What is the expected value of switching?

Answers

1. The expected value is 12.25. We have the following outcomes and probabilities:

Outcomes	Probabilities
1	$\frac{1}{36}$
2	$\frac{2}{36}$
3	$\frac{2}{36}$
4	$\frac{3}{36}$
5	$\frac{2}{36}$
6	$\frac{4}{36}$
9	$\frac{1}{36}$
10	$\frac{2}{36}$
12	$\frac{4}{36}$
15	$\frac{2}{36}$
16	$\frac{1}{36}$
18	$\frac{2}{36}$
20	$\frac{2}{36}$
24	$\frac{2}{36}$
25	$\frac{1}{36}$
30	$\frac{2}{36}$
36	$\frac{1}{36}$

2. There are 1000 possibilities. Thus, if you play, there is a probability of $\frac{1}{1000}$. The outcome is \$500, the expected value is $\$500 \times \frac{1}{1000} + (-\$1) \times \frac{999}{1000} = -\frac{499}{1000}$.

3. If you switch, you have a 50% chance of getting a box with \$50, and a 50% of getting a box with \$200. Thus, the expected value is $\$50 \times \frac{1}{2} + \$200 \times \frac{1}{2} = \$125$.