

Math 222, Quiz 8

Name:

Instructions: Answer the following questions fully, showing work where necessary.

1) Solve the following equation, finding the particular solution by “guessing”:

$$y'' + 2y' + 1 = e^x$$

The complementary equation is  $y'' + 2y' + 1 = 0$ . The auxiliary equation to this is  $r^2 + 2r + 1 = 0$ . This factors as  $(r + 1)^2 = 0$ , so that the complementary equation is  $c_1e^{-x} + c_2xe^{-x}$ . Since 1 is not a root of the auxiliary equation, our “guess” should be  $Ae^x$ . Plugging this in, you should find  $A = \frac{1}{4}$ . Thus, the particular solution is  $\frac{1}{4}e^x$ .

2) Now solve the equation by finding the particular solution by variation of parameters (obviously, you don't need to find the complementary solution again).

We have  $y_1 = e^{-x}$  and  $y_2 = xe^{-x}$ . Thus, we have to solve the equations:

$$\begin{aligned}v_1'e^{-x} + v_2'xe^{-x} &= 0 \\v_1'(-e^{-x}) + v_2'(e^{-x} - xe^{-x}) &= e^x\end{aligned}$$

Solving this gives  $v_1' = -xe^{2x}$  and  $v_2' = e^{2x}$ . Then  $v_1 = -\frac{xe^{2x}}{2} + \frac{e^{2x}}{4}$  and  $v_2 = \frac{e^{2x}}{2}$ . Then  $y_p = v_1y_1 + v_2y_2 = \frac{e^x}{4}$ , as above.

Bonus (1 pt.): What book is currently number one on the New York Times Nonfiction best seller list?

I am America (and so can you!) by Stephen Colbert