A summary of the tests for series

We will consider a series of the form $\sum_{n=a}^{\infty} a_n$

Name	How to use it	When to use it	Does it tell you if the series converges?	Does it tell you if the series diverges?	Example
n-th term test	Calculate $\lim_{n\to\infty} a_n$	Always	No	Yes, if the limit is not equal to 0	$a_n = \frac{n}{n-1}$
Integral test	Calculate $\int_a^{\infty} f(x)dx$	When $f(x)$ is easy to integrate	Yes, if the integral converges	Yes, if integral diverges	$a_n = \frac{1}{n \ln n}$
Direct Comparison	$a_n \leq b_n$ or $a_n \geq b_n$	There is a $\cos n$ or $\sin n$ in the numerator	Yes, if it is less than a convergent series	Yes, if it is bigger than a divergent series	$a_n = \frac{\sin^2 n}{n^2}$
Limit comparison	Calculate $\lim \frac{a_n}{b_n} = L$	When a_n is a fraction of n to various powers	Yes, if $L \neq \infty$ and $\sum b_n$ converges	Yes, if $L \neq 0$ and $\sum b_n$ diverges	$a_n = \frac{n+1}{n^2(1+\sqrt{n})}$
Ratio test	Calculate $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \rho$	a_n has z^n and/or $n!$ in it	Yes, if $\rho < 1$	Yes, if $\rho > 1$	$a_n = \frac{3^n}{n!}$, power series
Root test	Calculate $\lim_{n\to\infty} a_n^{\frac{1}{n}} = \rho$	a_n has z^n, or $f(n)^n$ Also, $n^{\frac{1}{n}} \to 1$	Yes, if $\rho < 1$	Yes, if $\rho > 1$	$a_n = \frac{2^n 3^n}{n^n}$
Alternating series test	Show $u_{n+1} \leq u_n$ $u_n \geq 0$ and $u_n \to 0$	Whenever you see $(-1)^n$ or $(-z)^n$	Yes, if the three. conditions are met. Check for absolute or conditional convergence.	No	$a_n = \frac{(-1)^n 3n^2}{n^3+1}$