Math 240, Quiz 1

Name:

Circle One: T 12:05 T 2:25 R 12:05 R 2:25

Instructions: Answer all questions fully, showing work where necessary.

1) Show that $\neg p \to (q \to r)$ and $q \to (p \lor r)$ are logically equivalent.

Here is a truth table for $(\neg p \to (q \to r)) \leftrightarrow (q \to (p \lor r))$:

p	q	r	$q \to r$	$\neg p \to (q \to r)$	$p \lor r$	$q \to (p \lor r)$	$(\neg p \to (q \to r)) \leftrightarrow (q \to (p \lor r))$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	T	T	T	T	T
T	F	F	T	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	F	F	F	T
F	F	T	T	T	T	T	T
F	F	F	T	T	F	T	T

You could also do this without the biconditional at the end, but you would need to write something along the lines of "Because they have the same truth table, they must be equivalent".

2)Express each of these statemtns using quantifiers. Then form the negation of the statment so that no negation is to the left of a quantifier. Next, express the negation in simple English (Do not simply use the words "It is not the case that")

a) All dogs have fleas. Let $D(x)$ mean "x is a dog", and $F(x)$ mean "x has fleas." Then we would write this statement as $\forall x(D(x) \to F(x))$.

The negation is $\neg\forall x(D(x) \to F(x)) \equiv \exists x\neg(D(x) \to F(x)) \equiv (\exists x(D(x) \land \neg F(x)$. Of course, there are several other things this is equivalent to. The translation of this negation is "There exists a dog that does not have fleas."

b) There exists a pig that can swim and catch fish.

Let $P(x)$ mean "x is a pig", $S(x)$ mean "x can swim", and $F(x)$ mean "x can fish". Then we write this as $\exists x(P(x) \land S(x) \land F(x))$.

The negation is $\neg\exists x(P(x) \land S(x) \land F(x)) \equiv \forall x\neg(P(x) \land S(x) \land F(x))$. Again, there are numerous things this is equivalent to. One translation of this is "No pig can swim and fish".