Math 240, Quiz 1

Name:

Circle One: T 12:05 T 2:25 R 12:05 R 2:25

Instructions: Answer all questions fully, showing work where necessary.

1) Show that $(p \to q) \vee (p \to r)$ and$(p \to (q \vee r)$ are logically equivalent.

The truth table is this:

p	q	r	$p \to q$	$p \to r$	$(p \to q) \vee (p \to r)$	$q \vee r$	$p \to (q \vee r)$	$(p \to q) \vee (p \to r) \leftrightarrow (p \to (q \vee r)$
T	T	T	T	T	T	T	T	T
T	T	F	T	F	T	T	T	T
T	F	T	F	T	T	T	T	T
T	F	F	F	F	F	F	F	T
F	T	T	T	T	T	T	T	T
F	T	F	T	T	T	T	T	T
F	F	T	T	T	T	T	T	T
F	F	F	T	T	T	F	T	T

2)Express each of these statemtns using quantifiers. Then form the negation of the statment so that no negation is to the left of a quantifier. Next, express the negation in simple English (Do not simply use the words "It is not the case that")

a) There is a horse that can add.

Let $H(x)$ mean "x is a horse", and $A(x)$ mean "x can add". Then we write this as $\exists x(H(x) \wedge A(x)$.

The negation is $\neg\exists x(H(x) \wedge A(x)) \equiv \forall x(\neg H(x) \wedge A(x)) \equiv \forall x(\neg H(x) \vee \neg A(x)$. Of course, there are other things this is equivalent to. A correct translation might be "No horse can add".

b) Every koala can climb.

Let $K(x)$ mean "x can add", $C(x)$ mean "x can climb". Then we can interpret this statement as $\forall x(K(x) \to C(x))$

The negation is $\neg\forall x(K(x) \to C(x)) \equiv \exists x\neg(K(x) \to C(x)) \equiv \exists x(K(x) \wedge C(x))$. A proper translation might be "There exists a koala that cannot climb".