

Math 240, Quiz 3

Name:

Circle One: T 12:05 T 2:25 R 12:05 R 2:25

Instructions: Answer all questions fully, showing work where necessary.

1) Prove that the product of two rational numbers is rational.

Let x, y be rational (a side note: if x, y are rational, that means $x, y \in \mathbb{Q}$, not, as several people wrote, $x, y \in \mathbb{R}$). Then we can write $x = \frac{a}{b}$ and $y = \frac{c}{d}$, with $a, b, c, d \in \mathbb{Z}$, and $b \neq 0, d \neq 0$. Then $xy = \frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$, with $bd \neq 0$, and $ac, bd \in \mathbb{Z}$. Thus xy a rational number.

2) Determine whether these statements are true or false (and justify your conclusion).

a) $\emptyset \in \{\emptyset\}$

This is true. Many people were confused. Now, $\{\emptyset\}$ is NOT the empty set. It is the set that has exactly one element, that element being the empty set. \emptyset has no elements. $\{\emptyset\}$ has exactly one element. Since \emptyset is in fact a member of $\{\emptyset\}$, this statement is true.

b) $\{\emptyset\} \in \{\{\emptyset\}\}$ This statement is also true. Note that neither of these sets is the empty set.

c) $\{\{\emptyset\}\} \subseteq \{\{\emptyset\}, \{\emptyset\}\}$ This is true. In fact, the sets are equal (note that the right hand side has $\{\emptyset\}$ listed twice. Since we don't care about repetition, we could simply write the right hand side as $\{\{\emptyset\}\}$).