Math 240, Quiz 4

Name:

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Instructions: Answer all questions fully, showing work where necessary.

1)Determine whether the following function from $\mathbb{R}$ to $\mathbb{R}$ are one-to-one and/or onto.

$f(x) = -3x^2 + 7$ To see if it is one-to-one, we need to see that if $f(a) = f(b)$, then $a = b$. Thus we suppose $f(a) = f(b)$, or $-3a^2 + 7 = -3b^2 + 7$. Subtracting seven, we see that $-3a^2 = -3b^2$. Dividing by -3, we get $a^2 = b^2$. But this does not imply that $a = b$. For example, since $1^2 = (-1)^2$, but $1 \neq -1$ (in fact, $f(1) = 4 = f(-1)$. Thus, this function is not one-to-one. To see if it is onto, we let b be a real number, and see if we can find a such that $f(a) = b$. In other words, we want $-3a^2 + 7 = b$, or $-3a^2 = b - 7$, or $a^2 = \frac{b-7}{-3}$. But if $b - 7 > 0$, then the right side is negative. But $a^2 \geq 0$. So if $b - 7 > 0$, or $b > 7$, we have that $f(a) \neq b$ for any $a \in \mathbb{R}$. As an example, if $b = 8$, then we would need that $a^2 = -\frac{1}{3}$.

2) Find the least integer n such that $f(x)$ is $O(x^n)$ for each of these functions.

a) $f(x) = 2x^2 + x^3 \log x$

$2x^2$ is $O(x^2)$, while $x^3 \log x$ is $O(x^3 \log x)$. But $\log x < x$, so $x^3 \log x$ is $O(x^4)$. Thus, when we add these two functions, we get that $f(x)$ is $O(\max(x^2, x^4)) = 0(x^4)$. Thus, the answer is $n = 4$.

b) $f(x) = (x^3 + 5\log x)/(x^4 + 1)$

We can say the top is $O(x^3)$, while the bottom is $O(\frac{1}{x^4})$. So when you multiply the two functions together, you get $O(\frac{1}{x})$.