Math 240, Quiz 4

Name:

Circle One: T 12:05 T 2:25 R 12:05 R 2:25

Instructions: Answer all questions fully, showing work where necessary.

1)Determine whether the following function from $\mathbb{R}$ to $\mathbb{R}$ are one-to-one and/or onto.
$f(x) = x^5 + 1$

To see if it is one-to-one, we need to see that $f(a) = f(b)$ implies $a = b$. Suppose $f(a) = f(b)$. Then $a^5 + 1 = b^5 + 1$, or $a^5 = b^5$. Since 5 is odd, we can take the fifth root of both sides (note: we couldn't do this if the power were even). Thus $a = b$, and the function is indeed one-to-one. To see if it is onto, we need to see if any $b \in \mathbb{R}$, there exists an $a \in \mathbb{R}$ such that $f(a) = b$. So we want $a^5 + 1 = b$, or $a^5 = b - 1$. Since 5 is odd, we can take the fifth root without any problems (again, if the pwoer was even, then we would be in trouble, because $b - 1$ could be negative). Thus, we have $a = \sqrt[5]{b-1}$ is such that $f(a) = b$, so that the function is onto.

2) Find the least integer n such that $f(x)$ is $O(x^n)$ for each of these functions.
a) $f(x) = 3x^5 + (\log x)^4$

Well, $\log x < x$, so $(\log x)^4 < x^4$, so certainly $(\log x)^4$ is $O(x^4)$. Obviously, $3x^5$ is $O(x^5)$, so when we add the functions to get $f(x)$, we get $f(x)$ is $O(\max(x^5, x^4))$, which is $O(x^5)$. Thus, the answer is $n = 5$.

b) $f(x) = (x^4 + x^2 + 1)/(x^4 + 1)$
The top is $O(x^4)$, while the bottom is $O(\frac{1}{x^4})$. Thus, multiplying them, you get $O(\frac{x^4}{x^4}) = O(1)$. Thus, $n = 0$.