

Math 240, Quiz 8

Name:

Circle One: T 12:05 T 2:25 R 12:05 R 2:25

Instructions: Answer all questions fully, showing work where necessary.

1) Determine whether the relation R on the set of all integers is reflexive, symmetric, antisymmetric, and/or transitive, where $(x, y) \in R \leftrightarrow xy \geq 1$.

It is not reflexive ($(0, 0) \notin R$). It is symmetric, since multiplication is commutative. It is not antisymmetric ($(1, 2)$ and $(2, 1)$ are in R , but $1 \neq 2$). Finally, it is transitive: if $xy \geq 1$ and $yz \geq 1$, then $y \neq 0$, $x \neq 0$ and $z \neq 0$. So x and y have to have the same sign, as do y and z . Thus, x and z have the same sign, so xz must be positive, so $xz \geq 1$.

2) Let R be the relation represented by the matrix

$$\mathbf{M}_R = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Find R^{-1} and R^2

$$\mathbf{M}_{R^{-1}} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 1 \end{bmatrix}, \text{ while } \mathbf{M}_R = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

3) Let R be the relation $\{(a, b) | a \neq b\}$ on the set of integers. What is the reflexive closure of R ?

Since we are simply adding in the set of all $\{(a, a) | a \in \mathbb{Z}\} = \{(a, b) | a = b\}$, we get that the reflexive closure is the set of pairs of integers, or $\mathbb{Z} \times \mathbb{Z}$.

Bonus (1 pt): How many states does Wisconsin border?

Four: Michigan, Minnesota, Iowa, and Illinois.