Abstract.

**Lecture one by X. Gong.** The following is a classical theorem of Kellogg. Let $\Omega$ be a bounded domain in the real plane. Let $f$ be a function on the boundary. Assume that the boundary of $\Omega$ and $f$ are in the Hölder class $C^{k,\alpha}$. Then the harmonic function $u$ with boundary value $f$ is of class $C^{k,\alpha}$ on the closure of $\Omega$. Here $k > 0$ is an integer and $0 < \alpha < 1$.

The theorem has an immediate consequence on the boundary regularity of the Riemann mapping of a simply connected domain in the complex plane, another theorem of Kellogg.

We will outline Kellogg’s original proof via Fredholm’s method. Kellogg’s proof does not yield the exact regularity stated as above. Nowadays, there are other approaches to obtain the exact regularity. We will present one by modifying Kellogg’s argument slightly.

The results stated in the first talk will be used in the second talk on our recent joint work on common boundary value of holomorphic functions.

**Lecture two by F. Bertrand.** The following result will be presented. Let $X_1, X_2$ be two $C^\infty$ complex structures defined on the closures of two disjoint domains $\Omega_1, \Omega_2$. Suppose that both are small perturbation of the standard complex structure. Assume that the boundaries of two domains share a $C^\infty$ curve $\gamma$. For $j = 1, 2$, let $f_j$ be a holomorphic function on $\Omega_j$ with respect to $X_j$. If $f_1, f_2$ have the same continuous boundary value on $\gamma$, the boundary value must be $C^\infty$ too.

Note that the result fails easily for harmonic functions via a jump formula. It remains to be studied for holomorphic functions in higher dimensions.

A more precise version in Hölder spaces will be given. The result on common boundary values of holomorphic functions will be applied to study the regularity of the solution to an integro-differential equation for the Cauchy-Green operator in one variable.