Complex Analysis Seminar.

Two lectures: “Uniqueness for differential inequalities and applications to $J$-holomorphic curves.”, by Jean-Pierre Rosay.

These two (instead of just one) lectures will be given at a pace of a working seminar.

Among the first uniqueness results that come to mind, there are the unique analytic continuation Theorem and in one variable the isolated zero Theorem for holomorphic functions, i.e. for functions that satisfy the Cauchy-Riemann equation $\partial f = 0$. There is also the uniqueness result for O.D.E.’s with Lipschitz condition.

To keep with one complex variable, one can investigate what happens when the equation $\frac{\partial f}{\partial z} = 0$ is replaced by differential inequalities such as:

(i) $\left| \frac{\partial f}{\partial z} \right| \leq C|f|,$

or

(ii) $\left| \frac{\partial f}{\partial z} \right| \leq \epsilon |\frac{\partial f}{\partial \bar{z}}|.$

Such inequalities arise naturally in the study of $J$-holomorphic discs in almost complex manifolds, where $f$ is not scalar valued but vector valued. We should also discuss the relationship with quasi conformal mappings, and linearity versus non-linearity.

Inequality (i) is fairly easy to use. Inequality (ii) is more difficult. For inequality (ii), unlike for (i), the vector valued case is very different from the scalar valued case, and one gets easy but non trivial examples of non uniqueness. That question of scalar versus vector valued, should be discussed in more detail.

However, uniqueness results can be obtained for a strengthened version of (ii) that is relevant to the theory of $J$-holomorphic curves. These uniqueness results are somewhat surprising in view of immediate counterexamples to closely related problems.

The proofs rely on very simple weighted $L^2$ estimates and on the standard technique of Carleman estimates in P.D.E.’s. No familiarity of the audience with either weighted $L^2$ estimates nor with Carleman estimates will be assumed. That will likely be the topic of the second lecture.