Wisconsin Emerging Scholars

Vector Calculus: Math 234

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Now that you know each other's names...

Mathematics.

What is mathematics?

Why do some people hate it and others love it?

MATHEMATICS (II)

What is mathematics? Is it problem solving? A big never-ending puzzle? Or just a bunch of equations that we practice solving quickly?

Why do some people hate it and others love it? Is it because some are "good at math" and others are "bad at math?"

MATHEMATICS (III)

Here's my answers:

•What is mathematics? It is what you make it! It could be:

1) Philosophically: Mathematics is the study of problem solving, and of calming and focusing the mind in order to do so.

2) Test of Memorization: Memorize the methods and equations necessary to solve particular problems.

• Which could you get the most out of?

Mathematics (IV)

• Why do some people hate it and others love it? Everyone finds a point when Mathematics is difficult. Then good study habits are needed to succeed.

1) If you devote some time and thought to a problem, mathematics is more enthralling than a good puzzle or a well-played strategy game.

2) If one relies purely on previously learned ideas, then instead of discovering you're doing busy work.

• Which is more fun?

1ST EXERCISE: WATCHING TV

- Picture yourself lounging in front of the TV. Magically and mysteriously the Pythagorean Theorem is quickly proved before your eyes.
- Did you understand it? Could you repeat the method? Could you apply the method to other cases? Did YOU make any discoveries?

Let's try it out!

Let a and b be the height and base of a right triangle, and c the Hypotenuse.

Theorem of Pythagoras: $a^2 + b^2 = c^2$.

So...

$c^2 = 4 \cdot \frac{ab}{2} + (a - b)^2$.

Simplifying, we get $c^2 = a^2 + b^2$.

I think we have time for one question.

LOOKING BACK - TEST OF COMPREHENSION. (I)

What is the area of the large square?

A) a^2

B) b^2

C) c^2

D) $(b-a)^2$

Looking Back - Test of comprehension. (II)

What is the area of the small square?

A) a^2

B) b^2

C) c^2

D) $(b-a)^2$

Looking Back - Test of comprehension (III)

Why is this construction correct?

Looking Back - Test of comprehension (IV)

Why is this construction correct?

- Placing one triangle at a time, why is there no overlap and why do they match up perfectly?
- Why are the outside angles right angles?

Looking Back - Test of comprehension (V)

Why is this construction correct?

- Placing one triangle at a time, why is there no overlap and why do they match up perfectly?
- Why are the outside angles right angles?
- Answer: consider the angles! $\alpha + \beta = 90°$

LOOKING BACK - TEST OF COMPREHENSION. (VI)

Why is this construction correct?

- Placing one triangle at a time, why is there no overlap and why do they match up perfectly?
- Why are the outside angles right angles?
- Answer: consider the angles! $\alpha + \beta = 90°$
- How could we have made sure that was understood?

Looking Back - Test of comprehension (VII)

- Have me explain it better?
- Have you all ask questions?

LOOKING BACK - TEST OF COMPREHENSION (VIII)

- Have me explain it better? Nobody is perfect - no matter what teacher you have, there will be typos or things left un-explained.
- Have you ask questions?
  Teacher: "There are no bad questions."
  Student response: "But we can't all ask questions!"

LOOKING BACK - TEST OF COMPREHENSION (IX)

- Have you ask questions?

  Teacher: "There are no bad questions."

  Student response: "But we can't all ask questions!"

- This last point really brings together my view of WES.

Class Understanding. (I)

How often are you confused in a typical math lecture:

A) Every few minutes

B) Several times a day

C) Once a day

D) Once a week

E) Once a month

Class Understanding. (II)

How often do you ask questions in a typical math lecture:

A) Every few minutes

B) Several times a day

C) Once a day

D) Once a week

E) Once a month

Class Understanding. (III)

When you ask questions in a typical math lecture (during or after lecture), who are they typically to:

A) Your buddy

B) The random person sitting next to you

C) The teacher

QUESTIONS.

Now, I hope you're convinced that asking questions is crucial, so how can we maximize the possibilities?

What are some difficulties?

DIFFICULTIES.

- "There isn't time for everyone to speak up and ask questions in class! How do I know if mine's important!"
- "Math ain't fun."
- "My brain shuts off after a while", or "I want to hang with my friends"
- "Its hard to work in groups - I work better alone."

POSSIBILITIES (I)

- "There isn't time for everyone to speak up and ask questions in class! How do I know if mine's important!"
- Groups: you can ask your friends and work together first, and then ask the TAs. This maximizes opportunities to ask questions, and minimizes confusion! If there's one thing you should take home from this, its to ASK QUESTIONS not only of me but of each other.

POSSIBILITIES (II)

- "Math ain't fun."
- Yes it is! Its like solving puzzles - how often do you solve Sodoku or Crossword puzzles, or play Chess?
- "My brain shuts off after a while", or "I want to hang with my friends"
- Make it a social activity. Hang out together in the library while doing your homework :) Set up a time at least once a week to do just that.
- "Its hard to work in groups - I work better alone."
- Working in groups is a valuable life skill, which everyone should develop. If you worry that you'll "have to explain things to others," you should realize that teaching is the best way to firm up your knowledge. I guarantee that sometimes you will need help yourself.

2ND EXERCISE: THINKING THROUGH PROBLEMS (I)

- Now picture yourself working in a group of three people, step-by-step, through the solution to the problem.

2ND EXERCISE: THINKING THROUGH PROBLEMS (II)

- Now picture yourself working in a group of three people, step-by-step, through the solution to the problem.
- Heck, why don't you all get into groups now so you can visualize it better!

2ND EXERCISE: THINKING THROUGH PROBLEMS. (III)

...and try to solve any of the following problems:

Let n be a positive integer.

1) Show that $1 + 2 + 3 + 4 + ... + n = \frac{n(n+1)}{2}$.

2) Show that $n(n + 1)$ is always even.

3) Show that $n(n + 1)(n + 2)$ is always divisible by 3.

4) Show that $\frac{n^3(n^2-1)}{8}$ is always an integer.

2ND EXERCISE: THINKING THROUGH PROBLEMS (IV)

Karl Gauss invented the theory of modulus to solve such problems.

If you’ve done much programming, you’ve surely run into them (denoted %).

2ND EXERCISE: THINKING THROUGH PROBLEMS (V)

3) Show that $n(n+1)(n+2)$ is always divisible by 3.

• The integers (...-2,-1,0,1,2,3,...) can be split into three sets,
$A = (... - 3, 0, 3, 6, ...)$, $B = (... - 5, -2, 1, 4, 7, 10, ...)$, and
$C = (..., -4, -1, 2, 5, 8, 11, ...)$.

• Gauss showed that instead of checking statement (3) for every integer, we can just check for one from each set A, B and C. Say 1,2 and 3.

• You've probably already checked that
$1(1+1)(1+2)$, $2(2+1)(2+2)$, and $3(3+1)(3+2)$
are divisible by 3. q.e.d.

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This would be a good time for you to consult with your groupmates to resolve miscomprehensions and pinpoint topics for me to clarify.

GROUPWORK.

Math is not the only difficult thing - working in groups is not always simple!

What are some difficulties involved in working in groups?

What are some ways to facilitate good group functioning?

GROUPWORK (II)

Working in groups isn't so cut and dry - its a learning experience just like mathematics is!

For now, here is one tip, which is important in understanding mathematics anyways:

Responsibilities in WES

You should come to WES prepared to think deeply and be helpful to your groupmates. You should:

- Have a good night's sleep. Be well fed.
- Have been to lecture and looked over your notes
- Have started on the relevant homework
- Be ready to talk, think, and ask questions!