

Puzzles

November 16, 2005

Instructions: Choose your own groups, of 2-4 people. Rafael can be in a group, but he counts as two people to this limit. The group that gets the most points wins.

Puzzle 1 Philosophers (variable points)

100 philosophers are standing in a line. Each one has a hat, either blue or yellow. Each philosopher can see every other hat, but not their own.

Each philosopher, one at a time, must say what color hat they have, or die a gruesome death.

If philosophers are allowed to discuss a strategy in advance, what is the best possible strategy? That is, how many are they likely to save with what probability?

(you get 2 points for the optimal answer, 1 point for a "good" answer)

Puzzle 2 Decimals (1 point)

Show that $\overline{.9} = 1$.

Hint: Let $x = \overline{.9}$, and compute $10x - x$ in two different ways.

Puzzle 3 Bridge Crossing (1 point)

A group of four people has to cross a bridge. It is dark, and they have to light the path with a flashlight. No more than two people can cross the bridge simultaneously, and the group has only one flashlight. It takes different time for the people in the group to cross the bridge:

Annie crosses the bridge in 1 minute, Bob crosses the bridge in 2 minutes, Volodia Mitlin crosses the bridge in 5 minutes, Dorothy crosses the bridge in 10 minutes. How can the group cross the bridge in 17 minutes?

Puzzle 4 Division

a) (1 point) Show that 3 divides a number exactly when the sum of the number's digits is divisible by 3. For example 78 is divisible by 3 because $7 + 8 = 15$ is (since $1 + 5 = 6$ is), but 136 is not since $1 + 3 + 6 = 10$ is not divisible by 3.

b) (1 point) Show that 9 divides a number exactly when the sum of the number's digits is divisible by 9.

c) (bonus 3 points) Show that 7 divides a number exactly when removing the last digit from the number, and subtracting double that digit from the resulting number, is divisible by 7. For example, 721 is divisible by 7 because $72 - 2 \times 1 = 70$ is divisible by 7.

Puzzle 5 Coins (variable points)

You have 12 coins, all but one of equal weight. You also have a balance scale, which will tell you which of two collections of coins is heavier.

If you want to figure out which coin is the heavier one, how many weighings do you need to do?

($\frac{1}{2}$ points: 5 weighings)

(1 point: 4 weighings)

(3 points: 3 weighings)

(5 points: 2 weighings)

Puzzle 6 Integers?

a) (1 point) Show that $\frac{n(n+1)}{2}$ is an integer for any integer n .

b) (1 point) Show that $\frac{n(n+1)(n+2)}{3}$ is an integer for any integer n .

c) (bonus 3 points) Show that $\frac{n^2(n^3-1)}{8}$ is an integer for any integer n .

Puzzle 7 Mathematical "Induction"

Let $S_n = 1 + 2 + 3 + 4 + \dots + n$.

a) (1 point) Fix a number k , and assume that $S_k = \frac{k(k+1)}{2}$. Show that $S_{k+1} = (k+1)(k+2)/2$.

b) (3 points) Now show that $S_n = \frac{n(n+1)}{2}$ for all positive integers n .