Lindelöf spaces, $D$-spaces, and selection principles

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We report on recent research in collaboration with Marion Scheepers and with Leandro Aurichi. Classical combinatorial strengthenings of Lindelöfness, namely the Menger and Rothberger properties, yield new insights into longstanding open problems in topology. For example,

**Theorem 1** [3]. If it is consistent there is a supercompact cardinal, it is consistent with GCH that all Rothberger spaces with points $G_δ$ have cardinality $\leq \aleph_1$, and that all uncountable Rothberger spaces of character $\leq \aleph_1$ have Rothberger subspaces of size $\aleph_1$.

**Theorem 2** [3]. Every Rothberger space with points $G_δ$ has cardinality less than the first real-valued measurable cardinal.

**Theorem 3** [1]. Menger spaces are $D$-spaces.

**Theorem 4** [2]. CH implies that if a $T_3$ space $X$ is either separable or first countable, and if $X \times Y$ is Lindelöf for every Lindelöf $Y$, then $X$ is a $D$-space.

**Definitions.**

- A space $X$ has the Rothberger (Menger) property if for each sequence $\{U_n : n < \omega\}$ of open covers of $X$ (each closed under finite unions), for each $n$ there is a $U_n \in U_n$ such that $\{U_n : n < \omega\}$ covers $X$.

- A space $X$ is $D$ if for each open neighborhood assignment $\{V_x : x \in X\}$ there is a closed discrete $D$ such that $\{V_x : x \in D\}$ covers $X$.

**References**


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