Lindelöf spaces, D-spaces, and selection principles

Franklin D. Tall

We report on recent research in collaboration with Marion Scheepers and with Leandro Aurichi. Classical combinatorial strengthenings of Lindelöfness, namely the *Menger* and *Rothberger* properties, yield new insights into longstanding open problems in topology. For example,

Theorem 1 [3]**.** *If it is consistent there is a supercompact cardinal, it is consistent with GCH that all Rothberger spaces with points G_δ have cardinality $\leq \aleph_1$, and that all uncountable Rothberger spaces of character $\leq \aleph_1$ have Rothberger subspaces of size $\aleph_1$.*

Theorem 2 [3]**.** *Every Rothberger space with points G_δ has cardinality less than the first real-valued measurable cardinal.*

Theorem 3 [1]**.** *Menger spaces are D-spaces.*

Theorem 4 [2]**.** *CH implies that if a T_3 space X is either separable or first countable, and if $X \times Y$ is Lindelöf for every Lindelöf Y, then X is a D-space.*

Definitions.

- A space X has the *Rothberger* (*Menger*) property if for each sequence $\{\mathcal{U}_n : n < \omega\}$ of open covers of X (each closed under finite unions), for each n there is a $U_n \in \mathcal{U}_n$ such that $\{U_n : n < \omega\}$ covers X.
- A space X is D if for each open neighborhood assignment $\{V_x : x \in X\}$ there is a closed discrete D such that $\{V_x : x \in D\}$ covers X.

References

[1] Aurichi, L. F. D-spaces, topological games and selection principles. In preparation.

[2] Aurichi, L. F. and Tall, F. D. Lindelöf spaces which are indestructible, productive, or D. In preparation.

[3] Scheepers, M., and Tall, F. D. Lindelöf indestructibility, topological games and selection principles. In preparation.

Franklin D. Tall
Department of Mathematics
University of Toronto
Toronto, Ontario
M5S 2E4
CANADA
e-mail address: f.tall@utoronto.ca