Calculus topics
Math 221

(1) Functions: review of the concept of function.
(2) Limits
• Both formal \( \epsilon - \delta \) definition and intuition.
• Properties and their use in the calculation of simple limits.
• Continuity.
• Limits of rational functions.
• Trig limits (\( \lim_{\theta \to 0} \frac{\sin \theta}{\theta} \), etc).
(3) Derivatives
• Limit definition, slope and tangent line.
• Derivative as rate of change and applications.
• Properties and how to compute them (product rule, chain rules, etc).
• Trig and other classical derivatives.
• Implicit differentiation and the derivative of the inverse. Derivatives of inverse trig functions.
• Applications of implicit differentiation (related rates problems).
(4) Graph sketching and optimization
• Intermediate and Mean Value Theorem. Increasing and decreasing functions.
• Stationary points. Using the first derivative to find extrema.
• Concavity, convexity and inflection points. Second derivative test.
• Asymptotes (horizontal and vertical).
• *Parametric curves (e.g. if \( x = f(t) \), \( y = g(t) \) then find the slope of the tangent).
(5) Exponential and logarithms
• Definitions and derivatives.
• Behavior as \( x \to \infty \) and comparison with exponent growth, limits involving \( \exp \) and \( \log \).
• Exponential growth and applications
• The hyperbolic functions.
(6) Integrals
• Riemann sum and the description of integral as area. Summation notation.
• The fundamental theorem of Calculus.
• Indefinite and definite integrals.
• The derivative of the integral.
• Basic methods of integration: antiderivative and \( u \)-substitution.
• Applications:
  – Areas between graphs.
  – Volumes of solids of revolution (plane slices, washers, shells).
  – Lengths of curves, including parametrized curves.
  – Density and its use in finding the mass, moment of inertia, etc.
  – Work of a force in 1D.