is to convey to his students not only what mathematicians know, but also what they do, and how, and why. This is a problem in full and honest self-revelation. And (as Raymond Chandler has remarked in another connection) honesty is an art.

A revised version of an address delivered at a conference on the teaching of college mathematics, in Katada, Japan, Sept. 6, 1964.

SEARCHING FOR MATHEMATICAL TALENT IN WISCONSIN

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Several years ago, Professor L. C. Young conceived the idea that The University of Wisconsin might conduct a mathematical problem competition similar to those which have met with great success in other countries, most notably, Hungary. Through Professor Young's efforts and enthusiasm the National Science Foundation became interested and agreed to support a pilot program. The task of designing and organizing the specifics of the program fell to a three-man committee consisting of Professor L. Fejes-Toth from Budapest, who was visiting at Madison for a year, Professor A. Beck, and the author, as chairman. The program consisted of sending four problem sets of five problems each to every student in the Racine high schools, every student taking a mathematics course in the Madison and Milwaukee schools, and ten copies to every other high school in the state. The purpose of the wide distribution in Racine was to see if we could interest the student who through lack of previous motivation had shown no inclination to excel in or study mathematics. Unfortunately, none of the students who actually submitted good papers fell into this category.

We attempted to pick problems which were solvable without any specific knowledge of course work, if the student was sufficiently clever. Of course, the students who had taken advanced mathematics courses might be able to use methods which they had learned in class or analogous methods, and thus find solutions more easily. The number of sophomores and juniors who did well, however, indicates some success in our problem choice. Among the top 46 students were 16 seniors, 20 juniors, 9 sophomores, and 1 freshman. Of these only four were girls.

In the Appendix is a copy of the letter which we sent with the first problem set to introduce the students to the competition. Also listed are the problems themselves, along with the number of papers received and the number of students who solved each problem completely. Every problem was solved by at least one student, most were solved by many students. Several students got all but one or two of the problems.

We were pleasantly surprised at the large response to the problems. In addition, letters from both students and faculty of the high schools indicate that there were many more students who actively worked and discussed the problems, but for any one of a number of reasons did not submit their results.

On the basis of the first three problem sets, the top forty-six students and
their teachers were invited to the University of Wisconsin campus for a one-day program. During the day the students heard an address by Professor P. C. Hammer on "The Role and Nature of Mathematics," had lunch at the home of Professor L. C. Young, Chairman of the Department of Mathematics, and met with Governor John W. Reynolds at the State Capital. There were also two sets of small group discussion sessions, in which about five students and one faculty member per group conferred.

Here again the response was excellent, with all but one of the students and many of the teachers accepting the invitation. At the dinner at the end of the program, each student was given a copy of the book *The Enjoyment of Mathematics* by Rademacher and Toeplitz.

The top five students, Raymond Hoffman, William I. Hibbard, Jeffrey Kuester, Michael Schwietzer, and Gerhard Weinhold were also given a copy of *What Is Mathematics?* by Courant and Robbins. Two of these five students were seniors, two were juniors, and Hibbard was a sophomore.

Comments by the teachers and students who attended this program lead us to believe that this year's program may get an even larger response in view of the general enthusiasm the visit to Madison has generated, and the enjoyable and profitable time had by those who attended. Also, we were told that because of the novelty of the program many students were shy about submitting their solutions.

The National Science Foundation has agreed to support the program for a second year. The new committee consists of Professor J. R. Smart as chairman, Professor D. W. Crowe, and the author.

**Appendix**

1. *Letter sent to students.*

"The National Science Foundation and the University of Wisconsin are co-operating in a search for students with the kind of imagination necessary to solve new and unusual problems. Because of the urgent need for people with this talent, we are hoping to discover them early and give them recognition, encouragement, and the opportunity for development. As a first step, we are circulating problems of a unique and challenging character, not requiring special knowledge from high school mathematics courses, but rather requiring ingenuity and insight. These problems may require considerable thought and experiment. There will be awards given at the end of the school year for the students with the best records.

We are including 5 problems on this first sheet. We will send out a new sheet each month, and also a sheet of solutions together with the names of the solvers. Send your methods of solution to: (Solutions due Saturday, December 21.)

Mathematical Talent Search

C/o Prof. L. C. Young, Chairman
Department of Mathematics
University of Wisconsin
Madison, Wisconsin

Good luck. I hope you enjoy the challenge of these problems."
2. Problems. The number after each problem indicates the number of students who solved that problem correctly. The problems are reproduced here precisely as they were sent to the students, even though we now realize several of them might have been worded better.

Problem Set 1—442 responses

1. Given \( n \) billiard balls numbered 1, 2, 3, \( \cdots \), \( n \), for some positive whole number \( n \), show that there are exactly two ways to arrange them in a circle so that the difference of the numbers on two adjacent balls is \( \pm 1 \) or \( \pm 2 \). Further, of the two arrangements, each is a mirror reflection of the other. (22)

2. Prove that the number of people who have danced with an odd number of partners is an even number of people. (13)

3. A man takes a walk on a flat plane according to the following pattern:
   a) he walks due north however long he wishes
   b) he walks in any direction except south however long he wishes
   c) he repeats steps a) and b) in that order as often as he wishes, as long as he does not cross his path.
   Prove that it is impossible for the man to return to the place where he started. (6)

4. If \( n \) is any positive whole number prove that 6 divides the product \( n(n+1)(n+2) \) and 24 divides \( n(n+1)(n+2)(n+3) \). Can you guess the largest number that will always divide the product \( n(n+1)(n+2) \cdots (n+k-1) \). (39)

5. Show that if a convex polygon \( P \) is symmetric about some point \( C \) then any inscribed triangle \( T \) has area less than or equal to half the area of \( P \). Can it ever be equal? (41)

Definitions: A polygon (or figure) is called \textit{convex} if for each two points on or inside the polygon (or figure) the line segment joining these two points is on or inside the polygon (or figure).

\begin{center}
\textbf{Convex} \hspace{2cm} \textbf{Not convex}
\end{center}

A polygon (or figure) is called symmetric about a center \( C \) if whenever a point \( P \) is in the polygon (or figure) so is the point \( Q \) which is the same distance from \( C \) as \( P \) but in the exact opposite direction through \( C \).

\begin{center}
\textbf{Not symmetric about any point} \hspace{2cm} \textbf{Symmetric about} \( C \)
\end{center}
Problem Set 2—247 Responses

1. There are \( N \) people in a room, and each one has shaken hands with at least one other person in the room. Show that there are at least two people who have shaken hands the same number of times. (64)

2. Is it possible to trace the accompanying figure without lifting the pencil from the paper and without retracing any lines? If so, show how; if not, show why not. (43)

3. In a certain club each member is on two committees and any two committees have exactly one member in common. There are five committees. How many members does the club have? (172)

4. Four houses, which form the vertices of a square, are to be linked by a shortest system of roads such that by traveling only on the roads it is possible to reach any house from any other. Show that the system of roads consisting of the diagonals is not the shortest one. (42)

5. Show that a number is divisible by 9 if and only if the sum of its digits is divisible by 9. (For example, 9 divides 1+8+7+2=18, so 9 divides 1872.) (26)

Problem Set 3—150 Responses

1. Show that in a group of \( N \) people, there are at least two who have shaken hands with the same number of other people in the group. (Note: This is identical to problem one of the second sheet, without the assumption that everyone shakes hands at least once.) (40)

2. A man walks due south ten miles, due west ten miles, and finds himself back at his starting point. Find all points at which he might have started in order to do this. (Assume the earth is spherical.) (30)

3. If \( a_1, a_2, a_3, \ldots, a_n \) are any \( n \) numbers, the arithmetic mean is defined to be \( (a_1 + a_2 + a_3 + \cdots + a_n)/n \) and the root mean square is \( [(a_1^2 + a_2^2 + \cdots + a_n^2)/n]^{1/2} \). Prove first that \( (a_1 + a_2)/2 \leq [(a_1^2 + a_2^2)/2]^{1/2} \), and using this, show that \( (a_1 + a_2 + a_3 + a_4)/4 \leq [(a_1^2 + a_2^2 + a_3^2 + a_4^2)/4]^{1/2} \). (24)

4. A circle of radius one is tangent to and inside of a circle of radius two; it rolls around the circumference of the larger circle so that they are always tangent. Pick any point \( P \) on the circumference of the smaller circle, and determine the shape of the path this point makes when the circle moves as described. Show that your answer is correct. (25)

5. Can a solid cube one foot on a side be constructed from bricks \( 2'' \times 4'' \times 8'' \)? Again, show that your answer is correct. (6)
1. In Problem No. 3, Set No. 3 it was shown that $(a+b)/2 \leq \sqrt{(a^2+b^2)/2}$ and
\[
\frac{a + b + c + d}{4} \leq \sqrt{\frac{a^2 + b^2 + c^2 + d^2}{4}},
\]
where $a$, $b$, $c$, and $d$ are arbitrary numbers. Show that
\[
\frac{a_1 + a_2 + a_3 + \cdots + a_k}{k} \leq \sqrt{\frac{a_1^2 + a_2^2 + \cdots + a_k^2}{k}},
\]
where $a_1$, $a_2$, $\cdots$, $a_k$ are arbitrary numbers, and $k$ is a positive integer. Hint: Consider the case $k=2^n$ first. (14)

2. If $x$ and $y$ are chosen to be integers, then $2x+6y$ is divisible by 19 if and only if $5x-4y$ is divisible by 19. (19)

3. A brick of dimensions $a \times b \times c$ is inscribed in a unit sphere. Let $\overline{PQ}$ be the shortest distance on the surface of the sphere between the points $P$ and $Q$. Let $S$ be the sum of all the distances $\overline{PQ}$ where the sum is taken over all possible pairs $P, Q$ of vertices of the brick. For what values of $a$, $b$, and $c$ is $S$ largest? (19)

4. In a group of $n$ boys and $n$ girls certain marriages are allowable. For each girl or boy there may be several allowable marriage partners. Show that it is possible to marry all of them to allowable partners only when for every integer $k \leq n$, every group of $k$ boys has a total of at least $k$ possible partners between them. (1)

5. An unending sequence $a_0, a_1, a_2, \cdots$, of positive integers is given. This sequence satisfies the conditions $a_0 < a_1 \leq a_2 + 20$, $a_1 < a_2 \leq a_1 + 20$, $a_2 < a_3 \leq a_2 + 20$, $\cdots$. Show that the unending decimal $b = .a_0a_1a_2 \cdots$ obtaining by writing the digits of $a_0, a_1, a_2, \cdots$ successively in order is not a repeating decimal; i.e., $b$ is an irrational number. (14)

MODERN COORDINATE GEOMETRY
A WESLEYAN EXPERIMENTAL CURRICULAR STUDY

HARRY SITOMER, Queens College, Flushing, N. Y.

During the summer of 1964, at Wesleyan University, Middletown, Connecticut, under a National Science Foundation grant, a team directed by Professor R. A. Rosenbaum, wrote a modern coordinate geometry text for high school students and a commentary for teachers.

The writing project developed directly out of a pilot experiment sponsored by the School Mathematics Study Group in 1961–62. In the experiment, teachers worked from a text based on Howard Levi’s “Foundations of Geometry and Trigonometry” (Prentice-Hall, 1956). All six teachers were unanimous in stating that the results were sufficiently encouraging to warrant the writing of a suitable text for this course.

The text written at Wesleyan University is now the course of study in five experimental centers throughout the country. Each center has a mathematics consultant and six teachers, one of whom acts as group chairman. Written reports are submitted regularly for each chapter, and comments are encouraged on text difficulty, suitability, and so on. This information is being collated, and will guide a rewrite committee during the summer of 1965.

“Modern Coordinate Geometry” uses only five axioms pertaining to geo-