CALCULUS (09/27/16)

1. Show that right now, there are two diametrically-opposed points on the earth's equator that are exactly the same temperature. (If this is too easy: are there two points on the equator that are separated by 120 degrees and have exactly the same temperature? Or even harder: Separated by $\sqrt{2}$ degrees and have the same temperature?)

2. (VTRMC'98,#1) Let

$$f(x,y) = \ln(1 - x^2 - y^2) - \frac{1}{(y-x)^2}$$

with domain $D = \{(x,y) : x \neq y, x^2 + y^2 < 1\}$. Find the maximal value of $f(x,y)$ over D.

3. (Putnam'89, #A2) Evaluate

$$\int_0^a \int_0^b e^{\max\{b^2x^2, a^2y^2\}}\, dy\, dx.$$

4. (Putnam'94, #A1) Suppose that a sequence $a_1, a_2, \ldots$ satisfies $0 < a_n \leq a_{2n} + a_{2n+1}$ for all $n \geq 1$. Prove that the series $\sum a_n$ diverges.

5. From last time: (Putnam'15, #B1): Let f be a three times differentiable function such that f has at least five distinct real zeros. Prove that

$$f + 6f' + 12f'' + 8f'''$$

has at least two distinct real zeros.

6. (Putnam'90, #B5) Is there an infinite series of non-zero real number $a_0, \ldots, a_1$ such that for any n, the polynomial

$$p_n(x) = a_0 + a_1x + a_2x^2 + \cdots + a_nx^n$$

has exactly n distinct real roots?

7. Bonus question: Look at my choice of problems. Can you guess what my favorite calculus theorems are?