PROBLEMS ABOUT INEQUALITIES
Wednesday, October 4

1. If $a, b, c > 0$, prove that $(a^2b + b^2c + c^2a)(ab^2 + bc^2 + ca^2) \geq 9a^2b^2c^2$.

2. If $a, b, c \geq 0$, prove that $\sqrt{3(a+b+c} \geq \sqrt{a} + \sqrt{b} + \sqrt{c}$.

3. Show that $\sqrt{a_1^2 + b_1^2} + \sqrt{a_2^2 + b_2^2} + ... + \sqrt{a_n^2 + b_n^2} \geq \sqrt{(a_1 + a_2 + ... + a_n)^2 + (b_1 + b_2 + ... + b_n)^2}$.

4. Find the minimum value of the function $f(x_1, x_2, ..., x_n) = x_1 + x_2 + ... + x_n$, where $x_1, x_2, ..., x_n$ are positive real numbers such that $x_1x_2...x_n = 1$.

5. (Putnam 2003) Let $a_1, a_2, ..., a_n$ and $b_1, b_2, ..., b_n$ be nonnegative real numbers. Show that

$$(a_1a_2...a_n)^{1/n} + (b_1b_2...b_n)^{1/n} \leq [(a_1 + b_1)(a_2 + b_2)...(a_n + b_n)]^{1/n}.$$

6. Which is larger, 2017^{2017} or 2018^{2016}?

7. (Putnam 1984) Find the minimum value of

$$(u - v)^2 + \left(\sqrt{2 - u^2} - \frac{9}{v}\right)^2$$

for $0 < u < \sqrt{2}$ and $v > 0$.

8. (Putnam 2004) Let m and n be positive integers. Show that

$$\frac{(m+n)!}{(m+n)^{m+n}} < \frac{m!}{m^m} \frac{n!}{n^n}$$