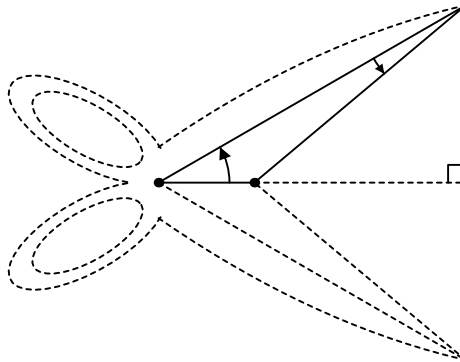


Practice Exam 1

1. Sketch the graph of a function $f(x)$ with domain $[0, 5]$ that is:
 - discontinuous at 1, but continuous from the negative side,
 - discontinuous from both sides at 3,
 - continuous everywhere else on the interval $(0, 5)$,
 - differentiable at 2 with $f'(2) = 1$, and
 - not differentiable at 4.
2. Evaluate $\lim_{x \rightarrow 0} \frac{(\sin x^2)^2}{x^2}$.
3. Let $y = \arctan \sqrt{x^2 + 2x}$. Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$. Recall that $\frac{d}{dx} \arctan x = \frac{1}{x^2 + 1}$. Hint: Simplify early and often.
4. The two blades of a pair of scissors are fastened at a point A (leftmost black dot). Let B be the point at the tip of the blade and D be the point where the blades meet. The distance from A to B is 20 cm; the angle between the line AB and the cutting edge of the blade BD is 5° , and the scissors are closing at a rate of 50 deg/sec. When the angle DAB is 30° , how fast is the paper being cut in cm/sec, that is, how fast is the distance from A to D increasing?



5. Suppose that $y^3 - x^2 = 4$.
- (a) Solve for y .
 - (b) From part (a), find $\frac{dy}{dx}$.
 - (c) From part (b), find $\frac{d^2y}{dx^2}$. Put things over a common denominator and simplify.
 - (d) Differentiate implicitly. Solve for $\frac{dy}{dx}$ in terms of x and y .
 - (e) Differentiate implicitly again. Substitute your expression for $\frac{dy}{dx}$ from part (c). Multiply through by y^3 . Solve for $\frac{d^2y}{dx^2}$ in terms of x and y .
 - (f) Substitute your answer to part (a) into your answer to part (e) and make sure it agrees with your answer to part (b).