1. First we compute:

$$\begin{aligned} r(t) &= e^t\langle 1, \sin t, \cos t\rangle \\ v(t) &= e^t\langle 1, \cos t + \sin t, \cos t - \sin t\rangle \\ |v(t)| &= e^t\sqrt{3} \\ a(t) &= e^t\langle 1, 2\cos t, -2\sin t\rangle \\ v(t)\times a(t) &= e^{2t}\langle -2, \cos t + \sin t, \cos t - \sin t\rangle \\ |v(t)\times a(t)| &= e^{2t}\sqrt{6} \end{aligned}$$

It is convenient to do part (c) before part (b):

(a) $T = \dfrac{v}{|v|} = \left\langle \dfrac{1}{\sqrt{3}}, \dfrac{1+\sqrt{3}}{2\sqrt{3}}, \dfrac{1-\sqrt{3}}{2\sqrt{3}} \right\rangle.$

(c) $B = \dfrac{v\times a}{|v\times a|} = \left\langle -\sqrt{\dfrac{2}{3}}, \dfrac{1+\sqrt{3}}{2\sqrt{6}}, \dfrac{1-\sqrt{3}}{2\sqrt{6}} \right\rangle.$

(b) $N = B\times T = \left\langle 0, \dfrac{1-\sqrt{3}}{2\sqrt{2}}, \dfrac{-1-\sqrt{3}}{2\sqrt{2}} \right\rangle.$

(d) $\kappa = \dfrac{|v\times a|}{|v|^3} = \dfrac{\sqrt{2}}{3e^{\pi/3}}.$

This is just one possible way to do the computation. You may have used different formulas or gotten different-looking answers that were actually the same as these.

(e) The direction of the desired line is $\langle 3, 1, -2\rangle \times \langle 2, 3, -1\rangle = \langle 5, -1, 7\rangle$, so

$$\frac{x-2}{5} = \frac{y+4}{-1} = \frac{z-5}{7}.$$

2. (a) Multiply through by ρ and substitute $\rho^2 = r^2 + z^2$ and $z = \rho \cos\phi$:

$$\begin{aligned} \rho &= 2\cos\phi \\ \rho^2 &= 2\rho\cos\phi \\ r^2 + z^2 &= 2z \\ r^2 + z^2 - 2z + 1 &= 1 \\ r^2 + (z-1)^2 &= 1 \end{aligned}$$

(b) Note that ϕ and φ are the same letter. Change to cylindrical coordinates to obtain $r^2 + (z - \frac{3}{2})^2 = \frac{9}{4}$ as in the previous part. If we fix θ, this is a circle of radius $\frac{3}{2}$ with center $r = 0, z = \frac{3}{2}$. Letting θ vary means spinning the circle around the z-axis, making a sphere with the same radius and center. The south pole is at the origin and the north pole is on the z-axis at $z = 3$.

3. (a) Since the mosquito is confined to the plane $y = 1$, it flies along the curve $z = 1 + x^3 + 12 = x^3 + 13$. The tangent line at $x = -2$, $z = 5$ has a slope $\dfrac{\partial z}{\partial x} = 3x^2 = 12$, so an equation for the line is $\dfrac{z-5}{x+2} = 12$. When the line intersects the yz-plane, $x = 0$, so $z = 29$. The desired point is thus $\langle 0, 1, 29 \rangle$.

(b) The mosquito is confined to the plane $y = 1$, so it never hits the xz-plane, where $y = 0$.

4. When $x \neq 3y^2$,

$$f(x,y) = \frac{x^2 - 9y^4}{y^2 - \frac{1}{3}x} = \frac{(x+3y^2)(x-3y^2)}{-\frac{1}{3}(x-3y^2)} = -3(x+3y^2),$$

but this expression makes sense everywhere and is continuous, so let

$$g(y) = -3(x + 3y^2) = -3(3y^2 + 3y^2) = -18y^2.$$