

Problem 1 If \vec{u} , \vec{v} , and \vec{w} are vectors and all the other quantities are scalars which of the following expressions make sense

1. $\vec{u} \cdot (\vec{v} \times \vec{w})$
2. $(\vec{u} \cdot \vec{v}) \times \vec{w}$
3. $\vec{u} \times \vec{v} + k$
4. $\vec{u} \cdot \vec{v} + k$
5. $(\vec{u} \times \vec{v}) \times \vec{w}$
6. $(\vec{u} \cdot \vec{v}) \cdot \vec{v}$
7. $(\vec{u} \cdot \vec{v})\vec{w}$

Problem 2 Show that for any two vectors \vec{u} and \vec{v} we have

$$|\vec{u} + \vec{v}|^2 + |\vec{u} - \vec{v}|^2 = 2(|\vec{u}|^2 + |\vec{v}|^2).$$

What geometric theorem about a parallelogram can you deduce from that?

Problem 3* Three vectors \vec{u} , \vec{v} , and \vec{w} . satisfy all the following properties:

$$|\vec{u}| = |\vec{w}| = 5, \quad |\vec{v}| = 1, \quad |\vec{u} - \vec{v} + \vec{w}| = |\vec{u} + \vec{v} + \vec{w}|.$$

If the angle between \vec{u} and \vec{v} is $\pi/8$, what is the angle between \vec{v} and \vec{w} .

Problem 4* Either show that the statements are true or give a counterexample. In each case $\vec{w} \neq \vec{0}$.

- (a) If $\vec{u} \cdot \vec{w} = \vec{v} \cdot \vec{w}$, then $\vec{u} = \vec{v}$.
- (b) If $\vec{u} \times \vec{w} = \vec{v} \times \vec{w}$, then $\vec{u} = \vec{v}$.
- (c) If $\vec{u} \cdot \vec{w} = \vec{v} \cdot \vec{w}$ and $\vec{u} \times \vec{w} = \vec{v} \times \vec{w}$, then $\vec{u} = \vec{v}$.