Worksheet 2

January 28, 2008

1. Consider the curve
$$\mathbf{r}(t) = \langle t, t^2, t^3 \rangle.$$
How would you compute the unit tangent $\mathbf{T}$, the unit normal $\mathbf{N}$, the binormal $\mathbf{B}$, the curvature κ, and the torsion τ "by the book"? Do not carry out the computataion, just describe what you would do.

2. What is $\mathbf{T} \times \mathbf{N}$? $\mathbf{N} \times \mathbf{B}$? $\mathbf{B} \times \mathbf{T}$?

3. If $\mathbf{v} = \mathbf{r}'$ is the velocity and $\mathbf{a} = \mathbf{r}''$ is the acceleration, what can you say about $\mathbf{v} \times \mathbf{a}$?

4. Based on your answers to problems 2 and 3, describe a better method for computing $\mathbf{N}$ and $\mathbf{B}$ than what you desribed in problem 1.

5. Carry out the computation from problem 1 using the method you just described. Why is your new method better?

6. Recall that
$$\frac{d\mathbf{T}}{ds} = \kappa \mathbf{N} \qquad \frac{d\mathbf{B}}{ds} = \tau \mathbf{N}.$$
(These may be taken as the definition of κ and τ, although they is not the best for computation.) You might expect that
$$\frac{d\mathbf{N}}{ds}$$
would give some new and interesting information. Using problem 2, show that this is not the case.