

# Worksheet 3

January 30, 2008

1. Show that the following are equivalent:

(a)  $|\mathbf{r}(t)|$  is constant.

(b)  $\mathbf{r}(t) \perp \mathbf{r}'(t)$ .

Interpret this in terms of a circle.

2. Show that the following are equivalent:

(a)  $|\mathbf{r}'(t)|$  is constant.

(b)  $\mathbf{r}'(t) \perp \mathbf{r}''(t)$ .

Interpret this in terms of a car driving on a curvy road.

3. Suppose that your speed is constantly 1. Show that the formula for curvature cleans up a lot.

4. Suppose that  $\mathbf{r}(t)$  is a curve and  $u(t)$  is an increasing function. Then  $\mathbf{r}(u(t))$  is another curve. What does it have to do with  $\mathbf{r}(t)$ ? To get started, consider  $u(t) = 2t$ .

5. Show that the unit tangent, unit normal, binormal, curvature, and torsion for the curve  $\mathbf{r}(u(t))$  at time  $t$  are the same as for the curve  $\mathbf{r}(t)$  at time  $u(t)$ .