Worksheet 8

February 18, 2008

1. Let $q = f(u, v, w)$, $u = 2x - y^2$, $v = x \sin 3y$, and $w = x^4$. Find $\dfrac{\partial q}{\partial x}$ and $\dfrac{\partial q}{\partial y}$. Your answer will involve f_u, f_v, and f_w.
2. Find the equation of the tangent plane to the surface $z = \sqrt{x} + \tan^{-1} y$ at the point $(9, 0, 3)$.
3. We say that a real-valued function of one variable $f(x)$ is *differentiable* at a point a if there is a number A and a function $g(x)$ such that

   $$f(x) = f(a) + A(x - a) + (x - a)g(x)$$

   and $g(x) \to 0$ as $x \to a$. In this case we call the number A the *derivative* of f at a and set $f'(a) = A$.

   (a) Relate this to the definition of $f'(a)$ that you know and love. Consider solving for $g(x)$.

   (b) Relate this to Prof. Caldararu's definition of a differentiable function of two variables.

   (c) Relate this to the Taylor series.

   (d) Relate this to the tangent line approximation.