

Worksheet 11

March 10, 2008

1. The U.S. post office will accept a box for shipment only if the sum of the length and the girth (distance around) is at most 108 inches.
 - (a) Find the dimensions of the largest acceptable box as you did last week.
 - (b) Find the dimensions of the largest acceptable box using Lagrange multipliers.
2. You are going to make a cylindrical can with a volume of 21.65625 cubic inches (= 12 fluid ounces).
 - (a) Find the dimensions of the can with the least surface area as you might have done in Math 221.
 - (b) Find the dimensions of the can with the least surface area using Lagrange multipliers.
 - (c) Why don't they make pop cans that shape?
3. Last week, you were going to make a 1-foot by 12-foot piece of metal into a gutter by folding x inches from each side up to an angle θ .
 - (a) Find the easiest possible way to find the values of x and θ that maximize the volume of the gutter.
 - (b) Write your solution nicely. I will collect it next week.
4. You are standing near a pond, looking at a fish in the water. Light travels in a straight line in air and underwater, but when it crosses the surface of the water, it bends. Let c_1 be the speed of light in air and c_2 the speed of light in water (both of these are less than the speed of light in vacuum, 3×10^8 m/s). Let θ_1 the angle the ray of light makes with the surface of the water on the air side and θ_2 the angle on the water side.
 - (a) Draw a picture.
 - (b) The light follows the "path of least time," that is, the time the light takes to travel from the fish to your eye is minimized. Using Lagrange multipliers, derive Snell's law, which is

$$\frac{\sin \theta_1}{c_1} = \frac{\sin \theta_2}{c_2} \quad \text{or} \quad \frac{\cos \theta_1}{c_1} = \frac{\cos \theta_2}{c_2}$$

depending on how you set it up.