Worksheet 13

March 24, 2008

1. Evaluate
$$\int_0^1 \int_x^1 \sin(y^2)\ dy\, dx.$$

2. (a) Let R be the portion of the unit disc lying in the first quadrant. Evaluate
$$\iint_R xy\ dA.$$

   (b) Let R be the portion of the unit disc lying in the upper half-plane. Evaluate
$$\iint_R xy\ dA.$$
   Why was the answer obvious?

   (c) Let R be the whole unit disc. Evaluate
$$\iint_R xy\ dA$$
   without working too hard.

3. Let R be the rectangle $a \le x \le b$, $c \le y \le d$. Let $f(x)$ and $g(y)$ be continuous functions. Explain why
$$\iint_R f(x)g(y)\ dA = \int_a^b f(x)\ dx \int_c^d g(y)\ dy.$$

4. Consider
$$\int_0^1 \int_0^{1-y^2} \int_0^y f(x,y,z)\ dz\, dx\, dy.$$
   (a) Describe the region of integration.
   (b) Express this as an integral in the order $dz\, dy\, dx$.
   (c) Express this as an integral in the order $dy\, dz\, dx$.

5. Given a continuous function f, let
$$g(x) = \int_0^x \int_0^y f(t)\ dt\, dy.$$
   That is, g is obtained by integrating f twice, starting the integration at 0. Show that g can be expressed as a single integral, namely
$$g(x) = \int_0^x (x-t)f(t)\ dt.$$