

Worksheet 17

April 16, 2008

1. Graph the curve $y = \cosh x$, $-1 \leq x \leq 1$. Then find its centroid. Recall that

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\cosh^2 x - \sinh^2 x = 1$$

$$\frac{d}{dx} \cosh x = \sinh x$$

$$\frac{d}{dx} \sinh x = \cosh x.$$

2. The function

$$E(k) = \int_0^{\pi/2} \sqrt{1 - k^2 \sin^2 t} \, dt \quad (0 < k < 1)$$

is called the *elliptic integral of the second kind*. Express the lengths of the following curves in terms of $E(k)$ for suitable k :

- (a) An ellipse with semimajor axis a and semiminor axis b .
- (b) The intersection of the sphere $x^2 + y^2 + z^2 = 4$ and the cylinder $x^2 + y^2 - 2y = 0$.