Worksheet 18

April 21, 2008

1. (a) For what values of b and c will the vector field

$$\mathbf{F} = \langle y^2 + 2czx,\ y(bx + cz),\ y^2 + cx^2 \rangle$$

   be the gradient of a function f?

   (b) What will f be in that case? How many possibilities are there?

   (c) Integrate $\mathbf{F}$ along the path $x = t$, $y = t$, $z = t$, $0 \leq t \leq 1$. Leave b and c general—don't plug in the values you found in part (a).

   (d) Integrate $\mathbf{F}$ along the path $x = t$, $y = t^2$, $z = t^3$, $0 \leq t \leq 1$.

   (e) Observe that if b and c take the values you found in part (a) then your answers to (c) and (d) equal $f(1, 1, 1) - f(0, 0, 0)$.

   (f) Argue that you could have answered part (a) by integrating $\mathbf{F}$ along a third path from $(0, 0, 0)$ to $(1, 1, 1)$, setting the answer equal to your answers to parts (c) and (d), and solving. Do it.

2. (a) Sketch the vector field

$$\mathbf{F} = \frac{\langle -y,\ x \rangle}{x^2 + y^2}.$$

   (b) Find the circulation

$$\oint \mathbf{F} \cdot \mathbf{T}\ ds$$

   of this vector field around various closed curves.

   (c) It is equivalent to integrate the differential form

$$\frac{x\ dy - y\ dx}{x^2 + y^2}$$

   around your closed curves. Understand how.

   (d) Your answers to part (b) should always be multiples of 2π. Speculate about what they mean.

   (e) Why is the "Component Test for Conservative Fields" on page 1164 not broken?