Worksheet 19

April 23, 2008

1. Find the simple closed curve that maximizes the line integral

$$\oint y^3\,dx + (3x - x^3)\,dy.$$

2. (a) Do problem §16.4 #29 if you haven't already.
   (b) Parametrize an ellipse of semimajor axis a, semiminor axis b, and center (c, d). Compute its area using Green's theorem. Half of your group should use $x\,dy$ and the other half $-y\,dx$, and you should compare your answers.
   (c) Compute the area of your favorite rose, both the old-fashioned way and using Green's theorem.

3. (a) From §16.4 #29, deduce the boxed formula on p. 1180.
   (b) What does $\frac{1}{2}(x\,dy - y\,dx)$ become in polar coordinates? (Hint: if $y = r\sin\theta$ then the product rule will give $dy = r\cos\theta\,d\theta + \sin\theta\,dr$.)

4. Read the discussion of the cycloid on page 710 to refresh your memory. Find the length of one arch of the cycloid and the area under it.