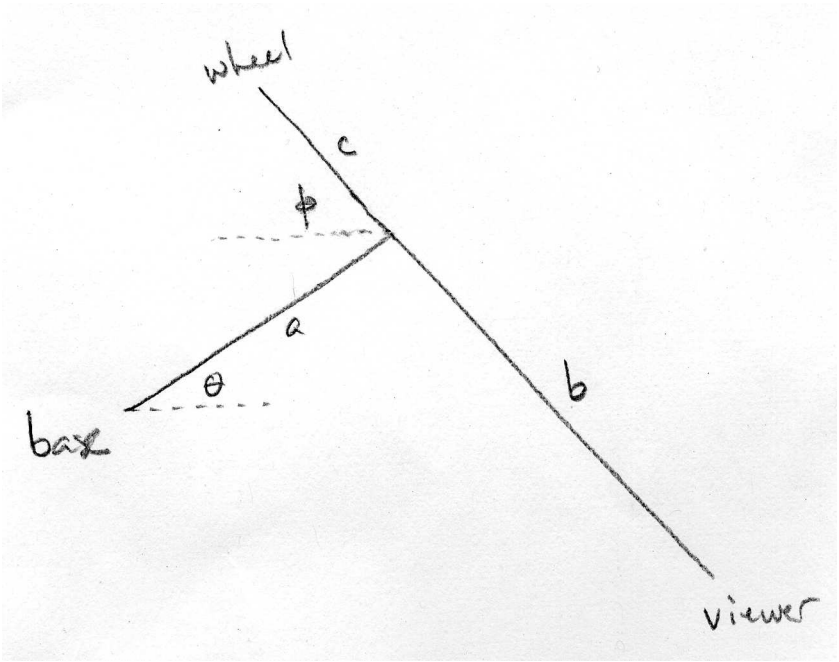
Worksheet 20

April 28, 2008

(This worksheet requires a planimeter.) We will refer to this diagram:



1. (a) Let the base be at the origin and the viewer at (x, y). Express x and y in terms of a, b, θ, and ϕ.
   (b) To find the area of the region whose boundary the viewer traces, you could integrate $\frac{1}{2}(x\,dy - y\,dx)$. Express this differential form in terms of a, b, θ, ϕ, $d\theta$, and $d\phi$. Simplify your answer using angle addition formulas.

2. (a) Now suppose the wheel is at (x, y) and is moved by a tiny amount $\langle dx, dy\rangle$. How far does it roll? (If it moves parallel to the arm, it will not roll. If it moves perpendicular to teh arm, it will roll a lot. Thus you should dot with a unit vector perpendicular to the arm.)
   (b) Express x and y in terms of b, c, θ, and ϕ.
   (c) Express you ranswer to part (a) in terms of b, c, θ, ϕ, $d\theta$, and $d\phi$.

3. (a) Argue that $\oint c\,d\phi = 0$ if the base is not in the middle of your region.
   (b) Similarly, argue that $\oint \cos(\theta + \phi)\,d(\theta + \phi) = 0$.
   (c) Compare the distance the wheel rolls to the area of the region whose boundary the viewer traces.