

## Worksheet 22

May 5, 2008

1. Consider the portion of the sphere of radius 1 between the  $xy$ -plane and the plane  $z = c$ , where  $0 \leq c \leq 1$ .
  - (a) Make a sketch.
  - (b) Find the volume.
  - (c) Find the area of the side.
2. Consider a circle in the  $xz$ -plane with radius  $r$  and center  $(R, 0)$ . The surface obtained by revolving this around the  $z$ -axis is called a *torus*. It looks like a donut or an inner tube.
  - (a) Make a sketch.
  - (b) Parametrize the circle with parameter  $\theta$ .
  - (c) Parametrize the torus with parameters  $\theta$  and  $\phi$ .
  - (d) Find its area.
3. The goal of this problem is to parametrize the hyperboloid of one sheet  $x^2 + y^2 - z^2 = 1$ .
  - (a) Sketch the circle  $x^2 + y^2 = 1$ . Parametrize it.
  - (b) Sketch the hyperbola  $x^2 - y^2 = 1$ . Parametrize it.
  - (c) Sketch the sphere  $x^2 + y^2 + z^2 = 1$ . Parametrize it.  
(Hint: you may want to write  $r^2 + z^2 = 1$  to emphasize the rotational symmetry.)
  - (d) Sketch the hyperboloid of one sheet  $x^2 + y^2 - z^2 = 1$ . Parametrize it.  
(Hint: you may want to write  $r^2 - z^2 = 1$  to emphasize the rotational symmetry.)