3.6 - Inverse Functions!

Two functions $f$ and $g$ are inverses of one another provided that

for each $x$ in the domain of $g$

and

for each $x$ in the domain of $f$

Say $g$ is the inverse of $f$: $g(x) =$

Say $f$ is the inverse of $g$: $f(x) =$

Note: This does NOT mean $\frac{1}{f(x)}$ and $\frac{1}{g(x)}$!

Example Are $f(x) = 4x - 1$ and $g(x) = \frac{1}{4}x + 1$ inverses?

Example Are $f(x) = 2x + 6$ and $g(x) = \frac{1}{2}x - 3$ are inverses?
Example Suppose that $f$ and $g$ are inverses.

- If $f(2) = 5$, then $g(5) =$

- If $g(7) = -1$, then $f(-1) =$

- If $f(a) = b$, then $g(b) =$

- The domain of $g$ is

- The range of $g$ is

The graph of $y = f^{-1}(x)$ is the graph of $y = f(x)$ reflected over
One-to-One Functions

A function is **one-to-one** if

**Examples:** Are these functions one-to-one?

\[ y = f(x) = 2x \quad \text{and} \quad y = f(x) = x^2 \]

\[ y = f(x) = \frac{1}{x} \quad \text{and} \quad y = f(x) = \lfloor x \rfloor \]
**The Horizontal Line Test**: A function $f$ is one-to-one if

For a function $f$ to have an inverse, look at the graph $y = f(x)$ and its “inverse”

![Graph of $y = f(x)$ and its inverse](image)

The function $f$ has an inverse function when

The graph of the “inverse” must
Finding $f^{-1}(x)$.

- $f(x) = 4x - 6$.

- $f(x) = x^2 - 4$
Finding $f^{-1}(x)$ cont.

- $f(x) = x^2 - 4$, domain $x \geq 0$.

- $f(x) = \frac{3x - 2}{x + 1}$. 