4.1 - Linear Functions

A **linear function** is a function defined by an equation of the form

where \( a \) and \( b \) are constants.

**Example.** Find the linear function satisfying:

- \( f(0) = 2, \ f(2) = 5 \)
- \( f(2) = 10 \), the graph \( y = f(x) \) is parallel to \( x - 2y = 4 \).
- The graph \( y = f(x) \) passes through \((1, 5)\) and \((-1, -3)\).
Example. You can buy a new car (a nice one) for $30,000, and after 5 years, you can sell it for $18,000. Assume the depreciation can be modeled by a linear function in terms of time.

- Find the linear function which models the depreciation, with $0 \leq t \leq 5$.

- Find the value of the car after 3 years.
Example. The distance (in mi) that a casino ship is from land after $t$ hours of taking off is modeled by the linear function

$$d(t) = 5 + 10t$$

• How fast is the ship going?

• How far are they after 3 hours?

• What is the $y$ intercept in the graph $y = f(x)$?

What does it mean?
Example. In 1917, Camp Randall was rebuilt with concrete stands to hold approximately 10,000 people. By 1951, the capacity of Camp Randall was 51,000. Assume that the stadium’s growth can be modeled by a linear function of time. Let $C(t)$ be the capacity of the stadium $t$ years after its inauguration in 1917.

Find $C(t)$.

Project how big the stadium would be in 2000.

The current capacity of Camp Randall has 80,321 seats. How good is the prediction?
Cost Functions and Marginal Cost

A function that gives the cost \( C(x) \) for producing \( x \) units of a commodity is called the

The additional cost to produce one unit is called the

**Example.** You open a business making cheesy macaroni art. The initial cost for starting up is $50 (the foldup table to set up on the corner of the road) and the marginal cost is $0.10 per artwork for materials.

- Find the linear function \( C(m) \), the cost for starting up shop and making \( m \) pieces of art.

- What is the cost of making 2000 macaroni pieces?
Example. The cost of making \( x \) whoopie cushions can be modeled by the linear function

\[ C(x) = 5000 + 0.15x \]

• Find the cost of making 1000 whoopie cushions.

• What is the marginal cost for making a whoopie cushion?

• Find the cost of making 1001 whoopie cushions.

• What is the \( y \)-intercept on the graph \( y = C(x) \)? What does it mean?