4.2 - Quadratic Functions

A **quadratic function** is a function of the form

\[ f(x) = ax^2 + bx + c \]

where \( a, b, \) and \( c \) are constants, and \( a \neq 0 \).

The graph of a quadratic function is called a

- When \( a > 0 \),
- When \( a < 0 \),

The turning point on the parabola is called the

The vertical line passing through the vertex is called the
Example. Graph the function using translation of $y = x^2$. Find the vertex, axis of symmetry, and intercepts.

$$y = x^2 - 6x + 8$$
The Graph of $y = ax^2$ Let $f(x) = \frac{1}{2}x^2$, $g(x) = x^2$, $h(x) = 2x^2$, and $j(x) = -2x^2$.

To summarize, $y = ax^2$ is a parabola, similar to $y = x^2$, and

- If $a < 0$, then the graph opens
- If $a > 0$, then the graph opens
- If $|a| > 1$, then the graph opens
- If $|a| < 1$, then the graph opens
Graph the following:

\[ y = (x - 4)^2 - 2 \]

\[ y = 3(x + 1)^2 + 4 \]

\[ y = -2x^2 - 4x + 9 \]

\[ y = \frac{1}{2}x^2 - 2x + 3 \]
**Extreme Values**

A quadratic function will have a maximum when

A quadratic function will have a minimum when

This will always happen at the vertex of the parabola.

Find the maximum / minimum output for the following functions:

- \( f(x) = x^2 - 4x + 3 \)

- \( f(x) = -2x^2 + 6x - 9 \)

- \( f(x) = 4x^2 + 8x + 3 \)
The Vertex Form of a Quadratic Function

The equation of the parabola \( y = ax^2 + bx + c \) can always be rewritten as

where the \( a \) is \( h \) and the \( b \) is \( k \).

**Example.** Find the quadratic function which passes through the point \((-2, 3)\) and has a vertex of \((1, 5)\).
**Example.** For what value of $c$ will the minimum value of $f(x) = x^2 - 4x + c$ be -7?

**Example.** For what value of $c$ will the maximum value of $f(x) = x^2 + 6x + c$ be 12?