5.6 - Compound Interest

Example. Suppose we invest $250 in a savings account which accrues 6% interest annually.

(a) How much interest is made after 1 year?

(b) How much money is in the account after 1 year?

<table>
<thead>
<tr>
<th>$t$ (years)</th>
<th>Amount in Account</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
</tr>
<tr>
<td>40</td>
<td></td>
</tr>
</tbody>
</table>

If an account accrues $r\%$ interest every year and started with $P$, the amount in the account after $t$ years is
Example. • If you invest $500 at 7% for 3 years, how much would you have?

• If you invest $500 for 3 years and end with $800, what is the interest rate?

• If you end up with $1000 dollars after investing for 4 years at 8%, what was the initial investment?

• If you invest $300 at 8% and yield $700, how long did you invest?

• If you invest $700 at 5% for 3 years, how much interest do you earn?
Compounding more than once a year

Example. An account gets 12% interest compounded quarterly. If you invest $200, how much is in the account after

- one quarter?

- four quarters (a year)?

- 10 years (40 quarters)?

An account with initial amount $P$ which gets $r\%$ interest compounded $n$ times per year will be worth

after $t$ years (or $t$ periods).
Compounding Continuously

Compounding continuously means that in a year, interest is computed

Fact that we will not prove (until calculus)

As $n \to \infty$, we get that

So the formula for continual compounding is

**Example.**  • If $150 is invested at 6% compounded continuously for 4 years, what is the yield?

• If you invest in an account compounding continuously, what rate would allow you to double in 8 years?
Effective Interest Rate

Example.

- Find the yield of investing $200 at 6% compounded monthly for 1 year.

- Find the yield of investing $200 at 6.16778% compounded annually for 1 year.

- Find the yield of investing $200 at 6% compounded monthly for 2 years.

- Find the yield of investing $200 at 6.16778% compounded annually for 2 years.

Given a nominal interest rate, the effective interest rate is

This is used to
Example. Which is better, investing at 6% compounded monthly, or 5.9% compounded continuously?