Math 241 practice problems

February, 2005

1. Consider the system of linear equations in 5 variables

$$\begin{bmatrix} 1 & 1 & -1 & 2 & 3 \\ -1 & 2 & 1 & 0 & 1 \\ 1 & 4 & -1 & 4 & 7 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ 8 \end{bmatrix}$$

Find all solutions to this system of equations, in the following form:

$$\vec{x} = \vec{x_0} + t_1 \cdot \vec{v_1} + \ldots + t_s \cdot \vec{v_s}$$

for a suitable integer s and suitable vectors $\vec{x_0}, \vec{v_1}, \ldots, \vec{v_s}$, so that the set of all solutions are in one-to-one correspondence with the set of all s-tuples $(t_1, \ldots, t_s)$.

2. Let A be the 2×2 matrix

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix}$$

Find a 2×2 matrix C such that $C^{-1} \cdot A \cdot C$ is a diagonal matrix.

3. (short answer) Find the inverse of the matrix

$$\begin{bmatrix} 1 & 4 & 5 \\ 0 & 2 & 6 \\ 0 & 0 & 3 \end{bmatrix}.$$

4. (short answer) Compute the determinant of the matrix

$$\begin{bmatrix} 0 & 2 & 2 & 0 \\ 1 & 0 & 3 & 0 \\ 0 & 5 & 0 & 5 \\ 7 & 15 & 8 & 8 \end{bmatrix}$$

5. (short answer) Let B be the 2×2 matrix

$$\begin{bmatrix} 1 & 2 \\ 2 & -2 \end{bmatrix}$$

Find the $(1, 1)$ entry of A^{10}.

6. Let B be the square matrix

$$\begin{bmatrix} 2 & 0 & 1 \\ -2 & 3 & 4 \\ -5 & 5 & 6 \end{bmatrix}$$

Then the trace of B^{-1} is equal to

A. 21

B. 13

C. -15

D. 0

E. -14

F. None of the above.

7. Let A be a 7×5 matrix. Which ones of the following statements are true?

(I) Every system of equations $A \cdot \vec{x} = \vec{0}$ has a non-zero solution.

(II) Every system of equations $\vec{y} \cdot A = \vec{0}$ has a non-zero solution. (Here $\vec{y}$ stands for the column vector with entries $y_1, \ldots, y_7$.

(III) If $\mathrm{rk}(A) = 5$, then every system of equations $A \cdot \vec{x} = \vec{b}$ has a solution.

(IV) If $\mathrm{rk}(A) = 5$, then every system of equations $\vec{y} \cdot A = \vec{c}$ has a solution.

(V) For every vector $\vec{b} \in \mathbb{R}^5$, the solutions of the system of equations $A \cdot \vec{x} = \vec{b}$ form a vector subspace of $\mathbb{R}^5$.

(VI) The system of equations $A \cdot \vec{x} = \vec{b}$ is consistent if $\vec{b}$ belongs to the span in $\mathbb{R}^7$ of the five column vectors of A.

8. Let A be the 4×5 matrix

$$A = \begin{bmatrix} 1 & 2 & -1 & 1 & 3 \\ 3 & 0 & 3 & -1 & x \\ 1 & -1 & 2 & -1 & 1 \\ 1 & -1 & -1 & 0 & 4 \end{bmatrix}$$

For which value of x does $\operatorname{rk}(A) = 2$?

A. $x = -3$

B. $x = 5$

C. $x = -1$

D. $x = 2$

E. $x = -4$

F. None of the above.

9. Let M be the following 4×4 matrix

$$M = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

There is a 4×4 matrix C such that

$$C^{-1} \cdot M \cdot C = \begin{bmatrix} \lambda_1 & 0 & 0 & 0 \\ 0 & \lambda_2 & 0 & 0 \\ 0 & 0 & \lambda_3 & 0 \\ 0 & 0 & 0 & \lambda_4 \end{bmatrix}$$

for suitable numbers $\lambda_1, \ldots, \lambda_4$. What is the value of $\lambda_1^2 + \lambda_2^2 + \lambda_3^2 + \lambda_4^2$?

A. 13

B. 21

C. 16

D. 25

E. 37

F. None of the above.

10. Let A be a 3×5 matrix such that $\mathrm{rk}(A) = 3$. Consider the following statements.

(I) The three rows of A are linearly independent.

(II) The five columns of A are linearly independent.

(III) There exists a 5×3 matrix B such that $A \cdot B = \mathrm{I}_3$.

(IV) There exists a 5×3 matrix C such that $C \cdot A = \mathrm{I}_5$. Then

A. Only (I) and (III) are true.

B. Only (II) and (III) are true.

C. Only (II) and (IV) are true.

D. Only (I), (III), (IV) are true.

E. (I), (II), (III), (IV) are all true.

F. None of the above.