Math 217 **Exam #2** **Spring 2011**

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STUDENT ID:

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TA:

Grading Table

Question	Possible Points	Points Earned
1	10	
2	15	
3	15	
4	15	
5	15	
6	15	
7	15	
EC	**10**	
Total	100	

Please read these instructions carefully before beginning.

1. Do not open the exam until 5:30. You will have 90 minutes to complete the exam.
2. Final answers must be written clearly in the proper space provided. No credit will be given for illegible or ambiguous answers. Answers with no justification will not be given credit.
3. No notes, calculators, or electronic devices are permitted.

1. (10 points) Evaluate the following Definite and Indefinite Integrals.

(a) $\int_{-\frac{\pi}{2}}^{0} \frac{\sin(w)}{(3+2\cos(w))^3} dw$

$u = 3 + 2\cos(w)$

$du = -2\sin(w)\,dw$

$$= -\frac{1}{2}\int_{-\frac{\pi}{2}}^{0} \frac{-2\sin(w)\,dw}{[3+2\cos(w)]^3} = -\frac{1}{2}\int_{3}^{5} \frac{1}{u^3}\,du = -\frac{1}{2} \cdot -\frac{1}{2}\, u^{-2}\Big]_3^5$$

$$= \frac{1}{4}\left(\frac{1}{5^2} - \frac{1}{3^2}\right) = \frac{1}{4}\left(\frac{1}{25} - \frac{1}{9}\right)$$

(b) $\int \frac{1}{\sqrt{x}(1+\sqrt{x})^2} dx \quad = 2\int \frac{1}{2\sqrt{x}(1+\sqrt{x})^2} dx$

$u = 1 + \sqrt{x}$

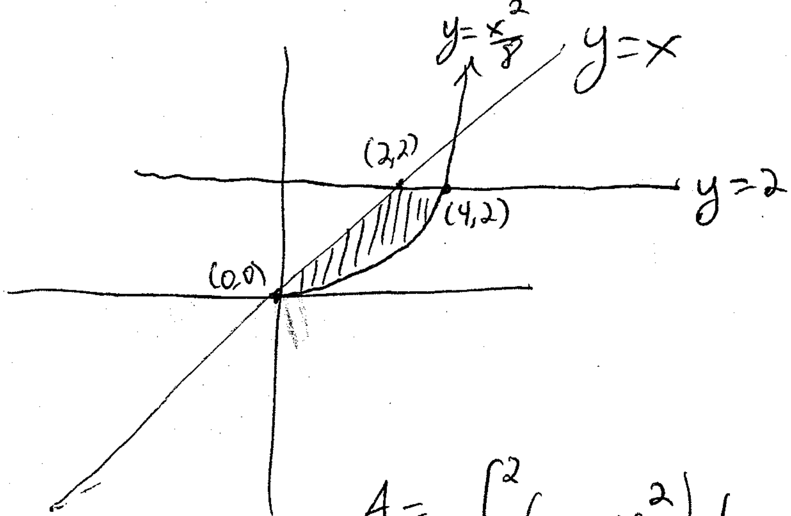
$du = \frac{1}{2\sqrt{x}} dx$

$$= 2\int \frac{1}{u^2} du$$

$$= -2u^{-1} + C$$

$$= -\frac{2}{1+\sqrt{x}} + C$$

2. (15 points) Find the area of the following region R in the plane. R is bounded on the left by $y = x$, R is bounded above by $y = 2$, and R is bounded below by $y = \frac{x^2}{8}$.



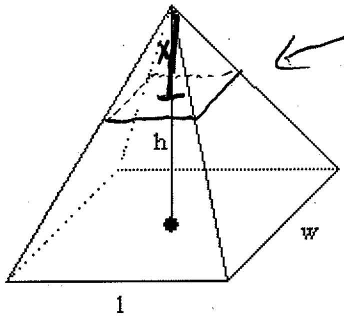
$$A = \int_0^2 \left(x - \frac{x^2}{8}\right)dx + \int_2^4 \left(2 - \frac{x^2}{8}\right)dx$$

$$= \left[\frac{x^2}{2} - \frac{x^3}{24}\right]_0^2 + \left[2x - \frac{x^3}{24}\right]_2^4$$

$$= \left[\left(2 - \frac{1}{3}\right) - 0\right] + \left[\left(8 - \frac{64}{24}\right) - \left(4 - \frac{1}{3}\right)\right]$$

$$= 6 - \frac{64}{24} = 6 - \frac{32}{12} = 6 - \frac{8}{3}.$$

3. (15 points) **Using an appropriate integral** (knowing a formula from geometry doesn't count), find the volume of a rectangular pyramid whose base is a rectangle with side-lengths l= 10 and $w = 20$ and with height $h = 5$.



A cut.

$w = 4h$

$l = 2h$

At the cut, the width = $4x$

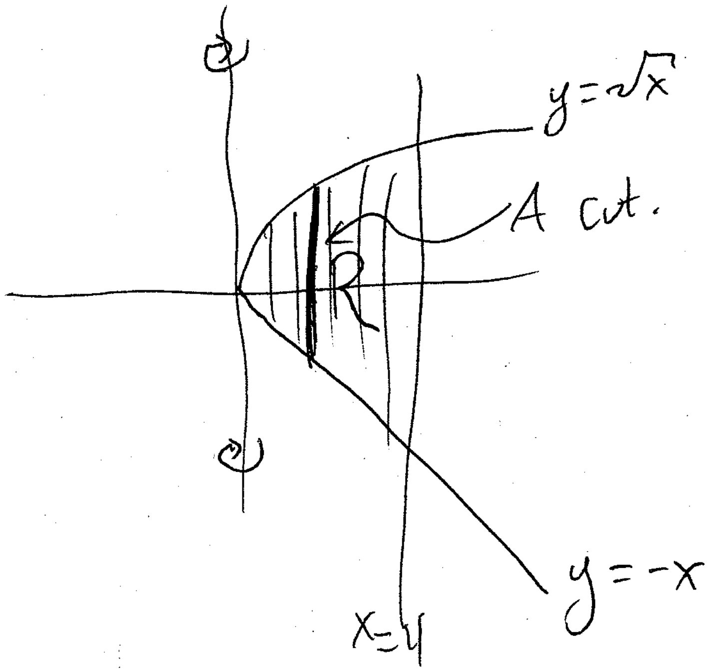
& the length = $2x$

$$\int_0^5 A(x)\,dx = \int_0^5 (\text{width of the cut})(\text{length of the cut})\,dx$$

$$= \int_0^5 (2x)(4x)\,dx = \int_0^5 8x^2\,dx$$

$$= \frac{8}{3}x^3\Big]_0^5 = \frac{8\cdot 125}{3} = \frac{1000}{3}.$$

4. (15 points) **Using the method of cylindrical shells**, find the volume of the solid generated by revolving the following region R about the y-axis. The region R is bounded by the curves $y = \sqrt{x}$, $y = -x$, and $x = 4$



$$\int_0^4 2\pi (\text{shell height})(\text{shell radius})\, dx$$

$$= \int_0^4 2\pi(\sqrt{x} + x)(x)\, dx$$

$$= 2\pi \int_0^4 (x^{3/2} + x^2)\, dx = 2\pi \left[\frac{2}{5}x^{5/2} + \frac{x^3}{3}\right]_0^4$$

$$= 2\pi\left[\frac{64}{5} + \frac{64}{3}\right]$$

5. (15 points)

(a) Assume that $\log_4(u) = 3.2$ and $\log_4(v) = 1.3$. Evaluate $\log_4(\frac{u^2}{16v^3})$

$$\log_4\left(\frac{u^2}{16v^3}\right) = \log_4(u^2) - \log_4(16v^3)$$

$$= \log_4(u^2) - [\log_4(16) + \log_4(v^3)]$$

$$= 2\log_4(u) - [2 + 3\log_4(v)]$$

$$= 2\cdot 3.2 - [2 + 3\cdot 1.3] = 6.4 - [2 + 3.9]$$

$$= 6.4 - 5.9 = .5.$$

(b) Find all numbers x that satisfy the following equation:

$$\frac{\log_{14}(13x)}{\log_{14}(4x)} = 2$$

Sol'n 1

$$\frac{\log_{14}(13) + \log_{14}(x)}{\log_{14}(4) + \log_{14}(x)} = 2$$

$$\log_{14}(13) + \log_{14}(x) = 2\cdot\log_{14}(4) + 2\log_{14}(x)$$

$$\log_{14}(13) - \log_{14}(16) = \log_{14}(x)$$

$$\log_{14}(x) = \log_{14}\left(\frac{16}{13}\right)$$

$$x = \frac{16}{13}.$$

Sol'n 2.

$$\log_{4x}(13x) = 2$$

$$(4x)^2 = 13x$$

$$x(16x - 13) = 0$$

$$x = 0$$

Not a sol'n, since $\log_{14}(13x)$ is $\log_{13}(0)$ is not defined

$$\boxed{x = \frac{13}{16}}$$

6. (15 points) Suppose you want to set aside some money now to pay for your future child's college tuition. You will invest in a savings account paying 6% interest per year compounded 4 times a year. You figure your child will go to UW and at current tuition levels, you will need about \$40,000. Assuming your child will go to college in 25 years, how much should you set aside now?

$$40000 = P_0 \cdot \left(1 + \frac{.06}{4}\right)^{4 \cdot 25}$$

$$P_0 = \frac{40000}{\left(1 + \frac{.06}{4}\right)^{100}}$$

7. (15 points) Evaluate the following:

(a) $\frac{d}{dx}\int_{\sin(x)}^{\cos(x)} \sec(t)dt$

$$= \sec(\cos(x)) \cdot [-\sin(x)] - \sec(\sin(x)) \cdot \cos(x).$$

(b) $\int_0^{2\pi} (\cos(x) + |\cos(x)|)dx$

$$= \int_0^{\frac{\pi}{2}} [\cos(x) + \cos(x)]dx + \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} [\cos(x) - \cos(x)]\, dx$$

$$+ \int_{\frac{3\pi}{2}}^{2\pi} [\cos(x) + \cos(x)]dx$$

$$= \int_0^{\pi/2} 2\cos(x)dx + \int_{\frac{3\pi}{2}}^{2\pi} 2\cos(x)dx$$

$$= 2\sin(x)\Big]_0^{\pi/2} + 2\sin(x)\Big]_{\frac{3\pi}{5}}^{2\pi}$$

$$= 2 + 2 = 4$$

Extra Credit (10 points) Prove algebraically (drawing a picture and pointing doesn't count) that if $f(x)$ is an odd function then $\int_{-a}^{a} f(x)dx = 0$ and that if $f(x)$ is an even function, then $\int_{-a}^{a} f(x) = 2\int_0^a f(x)dx$

If $f(x)$ is odd, that means

$f(-x) = -f(x)$.

If $f(x)$ is even, that means

$f(-x) = f(x)$.

$$\int_{-a}^{a} f(x)dx = \int_{-a}^{0} f(x)dx + \int_0^a f(x)dx$$

$u = -x$

$du = -dx$

$$= -\int_a^0 f(-u)\,du + \int_0^a f(x)dx$$

$$= \int_0^a f(-u)du + \int_0^a f(x)dx$$

So far so good.

If $f(x)$ is odd: $$= -\int_0^a f(u)du + \int_0^a f(x)dx = 0.$$

If $f(x)$ is even: $$= \int_0^a f(u)du + \int_0^a f(x)dx = 2\int_0^a f(x)dx.$$