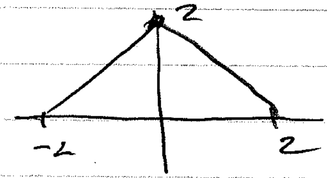
Math 217

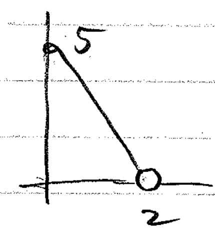
Homework 1

Feb 1

§4.1)

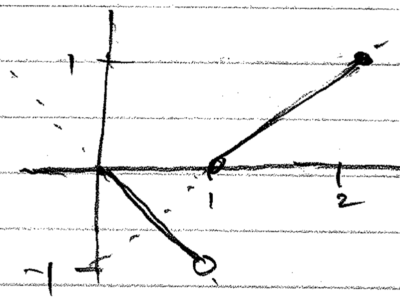
(8) 

Absolute maximum is 2 and occurs at 0.
Absolute minimum is 0 and occurs at -2 and 2.

(9) 

Global maximum is 5 and occurs at 0
There is no global minimum.

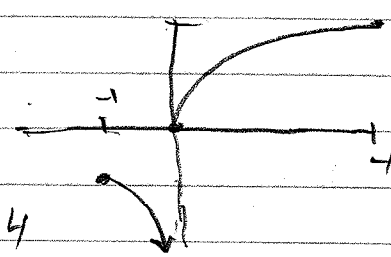
(17) $g(x) = \begin{cases} -x & \text{on } [0,1) \\ x-1 & \text{on } [1,2] \end{cases}$



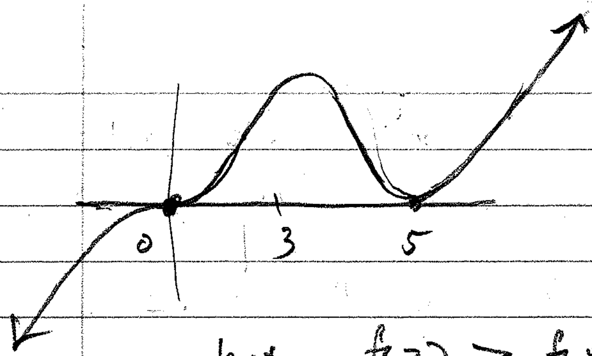
Global maximum is 1 at 2
There is no absolute minimum.

Theorem 1 requires g to be continuous, but it isn't so we do not expect the conclusion of the Theorem — that g must have a minimum — to be true. In this case g has a max but no min.

(18) $h(x) = \begin{cases} \frac{1}{x} & \text{on } [-1,0) \\ \sqrt{x} & \text{on } [0,4] \end{cases}$



h has a global max of 2 at 4 but explodes to $-\infty$ so it has no global minimum. h is not continuous at 0 so Theorem 1 does not apply.



$x^3(x-5)^2$ is odd

$x=5$ is either a turning point (like $x=0$ for $f(x)=x^2$) or it isn't (like 0 for $y=x^3$)

but $f(3) > f(5)$ so 5 is a local min.

$x=0$ is neither a local min or max

(since there are no other peaks, valleys, or "saddles").

(58) $y = \dfrac{x+1}{x^2+2x+2}$; $y' = \dfrac{x^2+2x+2 - (2x+2)(x+1)}{(x^2+2x+2)^2}$

$x^2+2x+2 = (x+1)^2+1 > 0$ for all x so y and y' are never undefined. Critical points occur when

$(x+1)^2+1 - 2(x+1)^2 = 0 \Rightarrow$

when $1 = (x+1)^2$

when $1 = \sqrt{(x+1)^2} = |x+1|$

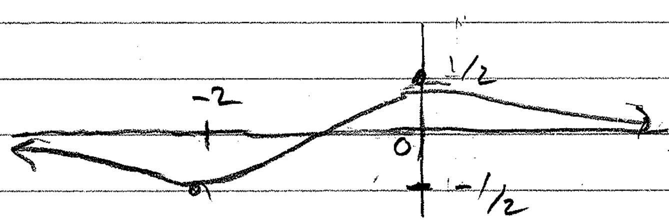
so when $x+1 = 1$ or $x+1 = -1$

$x = 0$ or -2

cp	f
0	$f(0) = 1/2$
-2	$f(-2) = -1/2$

From §2.6 $\lim\limits_{x\to -\infty} \dfrac{x+1}{x^2+2x+2} = 0 = \lim\limits_{x\to\infty} \dfrac{x+1}{x^2+2x+2}$

So 0 is the global max & -2 the global min.



74 (b) f Can have at most 2 local extreme values.

ex: $x(x-1)(x+1)$

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§4.2

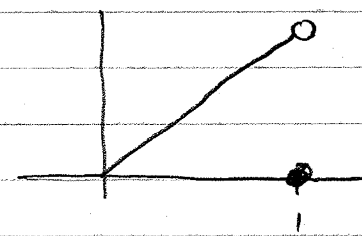
(3) $f(x) = x + \frac{1}{x}$ on $[\frac{1}{2}, 2]$

Want all x satisfying $f'(x) = \frac{f(2) - f(\frac{1}{2})}{2 - \frac{1}{2}} = 0$

$f'(x) = 1 - \frac{1}{x^2} = 0$ when $x^2 = 1$ and $x \in [\frac{1}{2}, 2]$

So only when $x = 1$ (discard $x = -1$)

(13) $f(x) = \begin{cases} x & \text{on } [0,1) \\ 0 & \text{at } 1 \end{cases}$



Rolle's Theorem requires
f to be continuous on $[0,1]$ (it isn't continuous at 1). We have no right to expect Rolle's Theorem to be valid here.

(14) $f(x) = \begin{cases} 3 & \text{at } 0 \\ -x^2+3x+a & \text{on } (0,1) \\ mx+b & \text{on } [1,2] \end{cases}$

MVT says (i) f must be continuous on $[0,2]$
(ii) differentiable on $(0,2)$

(i) we must ensure there is no jump discontinuity at the endpoints of each piece $(0,1)$, $(1,2]$.

Thus:

$$\lim_{x \to 0^+} -x^2+3x+a = a = 3$$

$$\lim_{x \to 1^+} mx+b = m+b = -1^2+3+3 = 5$$

(28) $\left.\begin{array}{l} f(0)=5 \\ f'(x)=2 \end{array}\right\}$ Must $f(x)=2x+5$? The answer is Yes.

Consider $f(x)-(2x+5)$, call it $g(x)$

Then $g'(x) = f'(x) - 2 = 2-2 = 0$

g is continuous (why?) and by MVT corollary 1 $g(x)$ is constant.

It remains to find this constant.

Notice $g(x) = g(0) = f(0) - 5 = 5-5 = 0$

for all x. so in fact $f(x) - (2x+5) \equiv 0$

so $f(x) \equiv 2x+5$

(30) if $f'(x) = c$ Then $f(x) = cx + d$ by the MVT.

This is Corollary 2 ... supply details

(32) (a) $y' = 2x$ $\quad (x^2)' = 2x$ so $y = x^2 + c$

(b) $y' = 2x-1$ $\quad (x^2-x)' = 2x-1$ so $y = x^2 - x + c$

(c) $y' = 3x^2+2x-1$ $\quad (x^3+x^2-x)' = 3x^2+2x-1$

so $y = x^3 + x^2 - x + c$

By MVT cor 2

(34) (a) $y' = \frac{1}{2\sqrt{x}}$ try $y=\sqrt{x}$, $y' = \frac{1}{2\sqrt{x}} \Rightarrow$

$y = \sqrt{x} + c$

(b) $y' = \frac{1}{\sqrt{x}}$ modify (a) by 2: $y = 2\sqrt{x} + c$

(c) $y' = 4x - \frac{1}{\sqrt{x}}$ try $2x^2 - 2\sqrt{x} + c$