Calculus 7.1

① Yes, it is one-to-one: no horizontal line intersects the graph more than once.

② No, for example $f(-1) = (-1)^4 - (-1)^2 = 1 - 1 = 0$,
$f(1) = 1^4 - 1^2 = 1 - 1 = 0$.

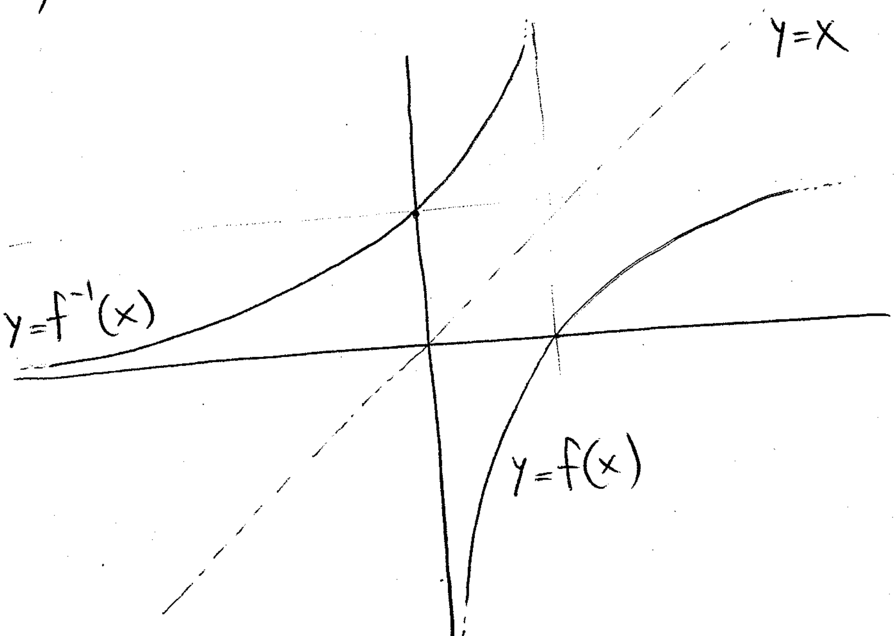
③ No, for example $f(-1) = 2 \cdot |-1| = 2$,
$f(1) = 2 \cdot |1| = 2$.

④ No, for example $f(0) = 0$,
$f(\frac{1}{2}) = 0$.

⑤ Yes, see ①.

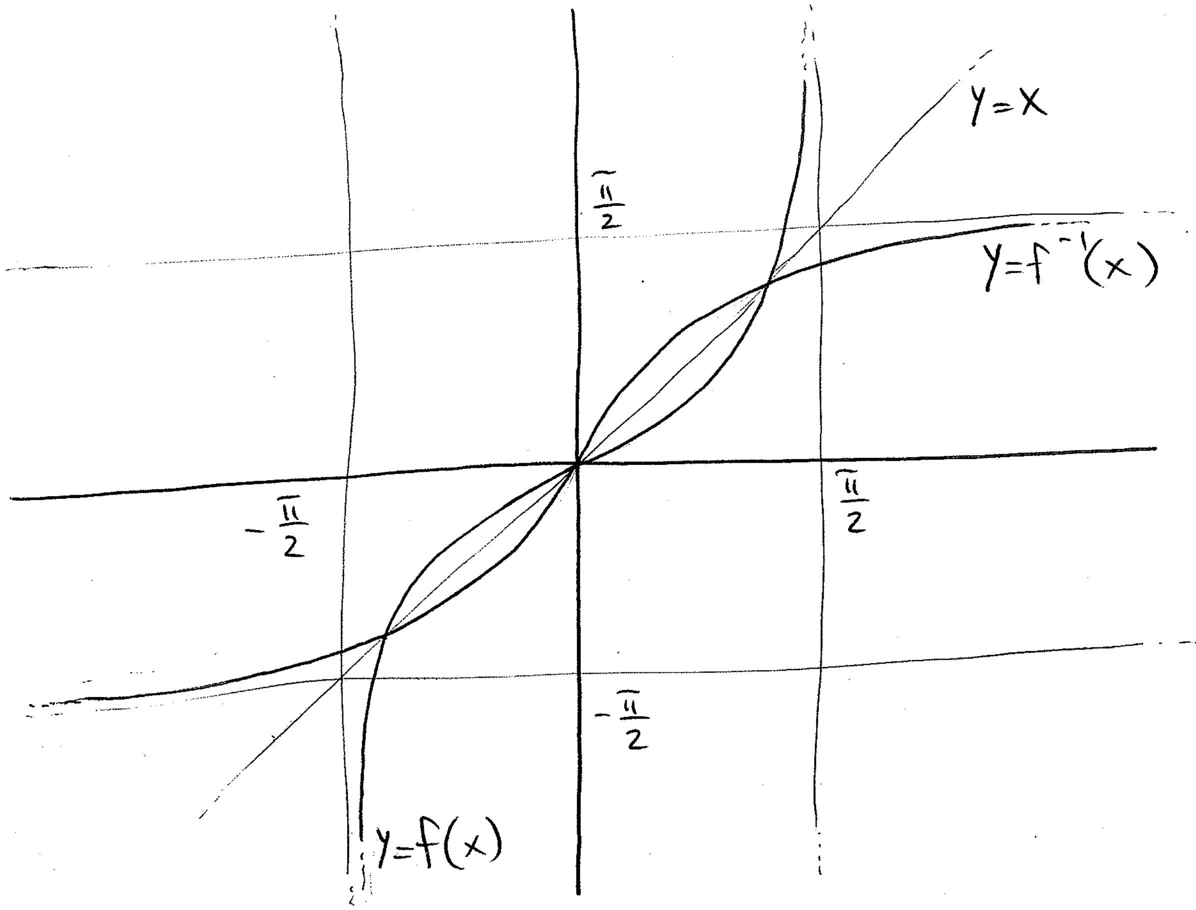
⑥ Yes, see ①.

⑫



Since (as we can see from the picture) domain of $f = (0, +\infty)$ and range of $f = (-\infty, 1)$, we have domain of $f^{-1} = (-\infty, 1)$ and range of $f^{-1} = (0, +\infty)$.

(14)



Since (as we can see from the picture)
domain of $f=(-\frac{\pi}{2},\frac{\pi}{2})$ and range of $f=(-\infty,\infty)$
we have
domain of $f^{-1}=(-\infty,\infty)$ and range of $f^{-1}=(-\frac{\pi}{2},\frac{\pi}{2})$.

(26) $y=f(x)=x^4$
By taking the $\frac{1}{4}$ power of both sides:
$y^{\frac{1}{4}}=x=f^{-1}(y)$ or equivalently $f^{-1}(x)=x^{\frac{1}{4}}$

domain of f^{-1} = range of $f^{-1}=[0,+\infty)$.

Check: $f(f^{-1}(x))=(f^{-1}(x))^4=(x^{\frac{1}{4}})^4=x$,

$f^{-1}(f(x))=f^{-1}(x^4)=(x^4)^{\frac{1}{4}}=x$.

(56) Yes, h will also be one-to-one.
To see this, assume that $h(x_1) = h(x_2)$.
This means that $\frac{1}{f(x_1)} = \frac{1}{f(x_2)}$.
By taking inverses of both sides, we get
$f(x_1) = f(x_2)$. But f is one-to-one by assumption,
so $x_1 = x_2$.

(58) Yes, because if g were not one-to-one, then
we could find x_1 and x_2 such that $g(x_1) = g(x_2)$
while $x_1 \neq x_2$, so that

$$f \circ g(x_1) = f(g(x_1)) = f(g(x_2)) = f \circ g(x_2).$$

Precalculus 3.1

(2) $8^{\frac{5}{3}} = \left(8^{\frac{1}{3}}\right)^5 = \left(\sqrt[3]{8}\right)^5 = 2^5 = 32.$

(6) $8^{-\frac{5}{3}} = \dfrac{1}{8^{\frac{5}{3}}} = \dfrac{1}{32}.$

(40) $x - 7\sqrt{x} + 12 = 0$

$(\sqrt{x})^2 - 7\sqrt{x} + 12 = 0,$ set $u = \sqrt{x}$

$u^2 - 7u + 12 = 0$

$(u-3)(u-4) = 0$

$u = 3 \longrightarrow \sqrt{x} = 3 \longrightarrow x = 9$

$u = 4 \longrightarrow \sqrt{x} = 4 \longrightarrow x = 16$

(48) $2^x = \dfrac{1}{3}$

Raise both sides to the power -4:

$(2^x)^{-4} = \left(\dfrac{1}{3}\right)^{-4}$

$2^{x\cdot(-4)} = 3^4$

$2^{-4x} = 81.$

Precalculus 3.2

② $\log_2 1024 = \log_2 2^{10} = 10$.

④ $\log_2 \frac{1}{256} = \log_2 \frac{1}{2^8} = \log_2 2^{-8} = -8$.

⑥ $\log_8 2 = \log_8 \sqrt[3]{8} = \log_8 8^{\frac{1}{3}} = \frac{1}{3}$.

⑭ $\log_8 2^{6.3} = \log_8 \left(8^{\frac{1}{3}}\right)^{6.3} = \log_8 8^{\frac{1}{3}\cdot 6.3} = \log_8 8^{2.1} = 2.1$.

⑱ Just choose $t = 2^8 = 256$, since $\log_2 2^8 = 8$.

⑳ Just choose $t = 2^{-9} = \frac{1}{2^9} = \frac{1}{512}$, since $\log_2 2^{-9} = -9$.

㉖ $\log_b 64 = 18$ means that $b^{18} = 64$.

To get b, raise both sides to the power $\frac{1}{18}$:

$$(b^{18})^{\frac{1}{18}} = (64)^{\frac{1}{18}}$$

$$b = (2^6)^{\frac{1}{18}} = 2^{\frac{6}{18}} = 2^{\frac{1}{3}} = \sqrt[3]{2}.$$

㉚ $\log_4 (3x+1) = -2$ means that $4^{-2} = 3x+1$.

So $\frac{1}{4^2} = 3x+1$

$\frac{1}{16} = 3x+1$, multiply both sides by 16:

$1 = 48x + 16$

$48x = -15$

$x = -\frac{15}{48}$

(38) We know that in general $\log_b x$ and b^x are inverse functions of each other, for <u>any</u> b.
In particular, when $b=4.7$, the inverse function of $f(x)=4.7^x$ is $f^{-1}(x)=\log_{4.7} x$.

(42) This one is slightly more involved. Start with

$y=f(x)=5^x-3$

$y+3=5^x$

take $\log_5$ of both sides:

$\log_5(y+3)=\log_5(5^x)$

Since 5^x and $\log_5 x$ are inverses of each other:

$\log_5(y+3)=x$

So $f^{-1}(y)=x=\log_5(y+3)$, or if you prefer to call your variable x instead of y,

$f^{-1}(x)=\log_5(x+3)$.

(54) $y=f(x)=\log_{5x} 6$

$(5x)^y=(5x)^{\log_{5x}6}$

$(5x)^y=6$

Raise both sides to the power $\frac{1}{y}$:

$5x=6^{\frac{1}{y}}$

$x=\frac{1}{5}6^{\frac{1}{y}}$, so $f^{-1}(y)=x=\frac{1}{5}6^{\frac{1}{y}}$, or equivalently
$f^{-1}(x)=\frac{1}{5}6^{\frac{1}{x}}$.

(60) $\log_3(\log_2(n)) = 2$

Applying 3^x to both sides:

$$3^{\log_3(\log_2(n))} = 3^2$$

$$\log_2(n) = 9$$

Finally, apply 2^x to both sides:

$$2^{\log_2(n)} = 2^9$$

$$n = 512.$$