Precalculus 3.3

(10) Since 83.2 is in the interval [83, 84), we can conclude that $\log k$ has 84 digits.

(12) $\log(mn) = \log(m) + \log(n) = 41.3 + 12.8 = 54.1$

Since 54.1 is in [54, 55), we can conclude that mn has 55 digits.

(16) $\log\left(\frac{b}{a}\right) = \log(b) - \log(a) = 205.4 - 203.4 = 2$

This means that $\frac{b}{a} = 10^2 = 100$.

(28) $\log_4(u^3 v^4) = \log_4(u^3) + \log_4(v^4)$

$= 3\log_4(u) + 4\log_4(v)$

$= 3 \cdot 3.2 + 4 \cdot 1.3 = 9.6 + 5.2 = 14.8$.

(30) $\log_4\left(\frac{u^2}{v^3}\right) = \log_4(u^2) - \log_4(v^3) = 2\log_4(u) - 3\log_4(v)$

$= 2 \cdot 3.2 - 3 \cdot 1.3 = 6.4 - 3.9 = 2.5$.

(34) $\log_4\left(\frac{x+4}{x-2}\right) = 3$ means that $4^3 = \frac{x+4}{x-2}$

$64(x-2) = x+4$

$64x - 128 = x + 4$

$63x = 132$

$x = \frac{132}{63}$.

(38) $\log_9(13x) = 2\log_9(4x)$

$\log_9(13x) = \log_9((4x)^2)$

$\log_9(13x) = \log_9(16x^2)$

$13x = 16x^2$

$16x^2 - 13x = 0$

$x(16x - 13) = 0$

So either $x = 0$ or $x = \frac{13}{16}$.

(58) $\log\left(\frac{100}{x}\right) = \log(100) - \log(x) = 2 - \log(x)$.

(64) $\log_b(x) = \log_b\left(y \cdot \frac{x}{y}\right) = \log_b(y) + \log_b\left(\frac{x}{y}\right)$.

By moving $\log_b(y)$ to the left-hand side one gets the desired equality:

$\log_b(x) - \log_b(y) = \log_b\left(\frac{x}{y}\right)$.

Precalculus 3.4

(14) $f(x) = 4 \cdot 2^{5x} = 2^2 \cdot 2^{5x} = 2^{5x+2} = 2^{5(x+\frac{2}{5})}$

$= (2^5)^{x+\frac{2}{5}} = 32^{x+\frac{2}{5}}$

So $\log_b f(x) = x + \frac{2}{5}$, which has slope 1,

if we choose $b = 32$.

(18) Using the formula on page 263 with $t_0 = 0$, we get

$$\frac{5}{4}\cancel{p_0} = \cancel{p_0}\, 2^{(t-0)/69}$$

$\frac{5}{4} = 2^{\frac{t}{69}}$, so $\log_2\left(\frac{5}{4}\right) = \frac{t}{69}$

So $t = 69 \cdot \log_2\left(\frac{5}{4}\right)$.

(20) (a) Let $p(t)$ be the number of bacteria after t hours. We know that $p(0) = 100$. In t hours the number of bacteria increases by a factor of $3^{t/2}$, so

$$p(t) = 100 \cdot 3^{t/2}.$$

(b) ~~$p(6) = 100 \cdot 3^{6/2} = 100 \cdot 3^3 = 100 \cdot 27 = 2700$.~~

$p(1) = 100 \cdot 3^{\frac{1}{2}} = 100 \cdot \sqrt{3} \approx 100 \cdot 1.73 = 173$.

(22)

$$\$8000\left(1+\frac{0.07}{12}\right)^{100\cdot 12}$$

using the formula on page 268.

(24) We know that

$$P\left(1+\frac{0.06}{4}\right)^{20\cdot 4}=27,707$$

where P is the initial amount deposited.

So $P=\dfrac{27,707}{1.015^{80}}$.

(27) We know that

$$1000\left(1+\frac{r}{12}\right)^{12}=1040$$

where r is the annual rate of interest.

So $\left(1+\frac{r}{12}\right)^{12}=1.04$. By taking the 12-th root:

$1+\frac{r}{12}=1.04^{\frac{1}{12}}$, so $\frac{r}{12}=1.04^{\frac{1}{12}}-1$,

and finally $r=12\left(1.04^{\frac{1}{12}}-1\right)$.

(39) The APY (Annual Percentage Yield) is the interest rate that would yield the same final amount of money if the interest were simple rather than compound. In other words, we should have

$$\cancel{P}\left(1+\frac{r}{n}\right)^{n}=\cancel{P}(1+APY).$$

By subtracting 1 to both sides, one gets

$$APY=\left(1+\frac{r}{n}\right)^{n}-1.$$

Precalculus 4.3

(2) (a) $\ln(x+y) = \ln(0.4+3.5) = \ln(3.9) \approx 1.36$

(b) $\ln(x) + \ln(y) = \ln(0.4) + \ln(3.5) \approx -0.92 + 1.25 = 0.33$

So they are <u>different</u>!

(4) (a) $\ln(xy) = \ln(1.1 \cdot 5) = \ln(5.5) \approx 1.7$.

(b) $\ln(x)\ln(y) = \ln(1.1)\ln(5) \approx 0.1 \cdot 1.7 = 0.17$

So they are <u>different</u>!

(14) By applying e^x to both sides, we get:

$$e^{\ln(2r^2-3)} = e^{-1}$$

$$2r^2 - 3 = e^{-1}$$

$$r^2 = \frac{e^{-1}+3}{2}, \quad \text{so} \quad r = \pm\sqrt{\frac{e^{-1}+3}{2}}.$$

(16) By taking ln of both sides, we get:

$$\ln(e^{4y-3}) = \ln(5).$$

$$4y - 3 = \ln(5)$$

$$4y = \ln(5) + 3$$

$$y = \frac{\ln(5)+3}{4}.$$

(20) $\ln(x+4)+\ln(x+2)=2$

$\ln((x+4)(x+2))=2$

$\ln(x^2+4x+2x+8)=2$

$\ln(x^2+6x+8)=2$

$e^{\ln(x^2+6x+8)}=e^2$

$x^2+6x+8=e^2$

$x^2+6x+8-e^2=0$

By using the quadratic formula (since $36-4(8-e^2)>0$):

$$x=\frac{-6\pm\sqrt{36-4(8-e^2)}}{2}.$$

(26) Because $\ln(c)=\text{area}(\frac{1}{x},1,c)=3$, we see that $c=e^3$.

(28) Since e^x is increasing, we only need to minimize $t^2+8t+3=f(t)$. We have $f'(t)=2t+8$.

$2t+8>0 \leftrightarrow t>-4$

		-4	
f'	$-$		$+$
f	$\searrow$		$\nearrow$

Therefore it's clear that f has an absolute minimum at $t=-4$.

(44) (a) For $\ln(x+5)$ to make sense, we need $x+5>0$, so the domain of f is $(-5,+\infty)$.

(b) Since the range of $\ln$ is $(-\infty,+\infty)$, it is clear that the range of f will also be $(-\infty,+\infty)$.

(c) $y=3+\ln(x+5)$

$y-3=\ln(x+5)$

$e^{y-3}=x+5$

$f^{-1}(y)=x=e^{y-3}-5$, or equivalently $f^{-1}(x)=e^{x-3}-5$

Check: • $f(f^{-1}(x))=f(e^{x-3}-5)=3+\ln(e^{x-3}-5+5)$

$=3+\ln(e^{x-3})=3+x-3=x=I(x)$.

• $f^{-1}(f(x))=f^{-1}(3+\ln(x+5))$

$=e^{3+\ln(x+5)-3}-5=e^{\ln(x+5)}-5$

$=x+5-5=x=I(x)$

(d) By part (b), the domain of f^{-1} is $(-\infty,\infty)$.

(e) By part (a), the range of f^{-1} is $(-5,+\infty)$.