P 4.4)

34)

a) $x^t = (e^{\ln(x)})^t = e^{t \cdot \ln(x)}$.

first equality is properties of $\ln(x)$ and e

second is rules for exponentiation.

b) Let us look at the linearization of

x^t at $t=0$.

$$\frac{d\, x^t}{dt} = \frac{d\, e^{t \ln(x)}}{dt} = e^{t \ln(x)} \cdot \ln(x) = x^t \ln(x)$$

so at $t = 0$ $y(0) = x^0 = 1$

and our linearization (or tangent to x^t at $t=0$)

is $y = x^0 \ln(x) \cdot t + 1$ so

for $t \approx 0$

$x^t \approx \ln(x) \cdot t + 1$ or

$$\frac{x^t - 1}{t} \approx \ln(x).$$

P 4.5)

2)

$3000 \cdot e^{.07 \cdot 15} = 3000 \cdot e^{1.05} \cong 8572.96$

6)

$P = 200$

$P(3) = 224$ so $P(3) = 200 \cdot e^{r \cdot 3} = 224$

$200 \cdot e^{3r} = 224$

$e^{3r} = \frac{224}{200}$

$3r = \ln\left(\frac{224}{200}\right)$

$r = \frac{\ln\left(\frac{224}{200}\right)}{3} \cong 0.037$

12)

$P(5) = Pe^{r \cdot 5}$

at the same time

$P(5) = P \cdot (1 + 1.5) = P \cdot 2.5$

so $P \cdot e^{5r} = P \cdot 2.5$

$e^{5r} = 2.5$

$5r = \ln(2.5)$

$r = \frac{\ln(2.5)}{5} \cong 0.18$

20)

$P = 300$

$P(t) = 300 \cdot e^{0.05 \cdot t} = 2400$

$e^{0.05 \cdot t} = \frac{2400}{300}$

$e^{0.05 \cdot t} = 8$

$0.05 \cdot t = \ln(8)$

$t = \frac{\ln(8)}{0.05} \cong 41.6$

7.2)

3) a) $\ln(\sin(\theta)) - \ln\left(\frac{\sin(\theta)}{5}\right) =$

$= \ln\left(\frac{\sin(\theta) \cdot 5}{\sin(\theta)}\right) = \underline{\ln(5)}$

b) $\ln(3x^2 - 9x) + \ln\left(\frac{1}{3x}\right) = \ln\left(\frac{3x^2 - 9x}{3x}\right) = \ln(x-3)$

c) $\frac{1}{2} \ln(4t^4) - \ln(2) =$

$= \ln(2t^2) - \ln(2) = \ln\left(\frac{2t^2}{2}\right) = \ln(t^2) = 2\ln(t)$

4) a) $\ln(\sec(\theta)) + \ln(\cos(\theta)) = \ln(\sec(\theta)\cos(\theta)) = \ln(1) = \underline{0}$

b) $\ln(8x+4) - 2\ln(2) = \ln(4(2x+1)) - 2\ln(2) =$

$= \ln(4) + \ln(2x+1) - 2\ln(2) = 2\ln(2) - 2\ln(2) + \ln(2x+1) =$

$= \underline{\ln(2x+1)}$

c) $3\ln\left(\sqrt[3]{t^2-1}\right) - \ln(t+1) = \ln(t^2-1) - \ln(t+1) =$

$= \ln\left(\frac{t^2-1}{t+1}\right) = \underline{\ln(t-1)}$

6) $(\ln(kx))' = (\ln(k) + \ln(x))' = \underline{\frac{1}{x}}$

12) $(\ln(2\theta+2))' = (\ln(2) + \ln(\theta+1))' = \underline{\frac{1}{\theta+1}}$

24) $(\ln(\ln(\ln(x))))' = \underline{\frac{1}{\ln(\ln(x))} \cdot \frac{1}{\ln(x)} \cdot \frac{1}{x}}$

22) $\left(\frac{x\ln(x)}{1+\ln(x)}\right)' = \frac{(\ln(x)+1)\cdot(1+\ln(x)) - \frac{1}{x}(x\ln(x))}{(1+\ln(x))^2} =$

$= 1 - \frac{\ln(x)}{(1+\ln(x))^2}$

28)

$$\left(\frac{1}{2}\ln\left(\frac{x+1}{1-x}\right)\right)' = \left(\frac{1}{2}\left(\ln(x+1) - \ln(1-x)\right)\right)' = \frac{1}{2}\left(\frac{1}{x+1} + \frac{1}{1-x}\right)$$

42)

$$\int_0^{\frac{\pi}{3}} \frac{4\sin(\theta)}{1-4\cos(\theta)}\,d\theta = \int_{-3}^{-1} \frac{du}{u} = \left[\ln(|u|)\right]_{-3}^{-1} = \ln(3) - \ln(1) = \ln(3)$$

$u = 1 - 4\cos(\theta)$

$du = 4\sin(\theta)\,d\theta$

$\theta = 0 \rightarrow u = -3$

$\theta = \frac{\pi}{3} \rightarrow u = -1$

46)

$$\int_2^{16} \frac{dx}{2x\sqrt{\ln(x)}} = \int_{\ln(2)}^{4\ln(2)} \frac{du}{2\sqrt{u}} = \left[\sqrt{u}\right]_{\ln(2)}^{4\ln(2)} = \sqrt{4\ln(2)} - \sqrt{\ln(2)} =$$

$$= 2\sqrt{\ln(2)} - \sqrt{\ln(2)} =$$

$$= \sqrt{\ln(2)}$$

$u = \ln(x)$

$du = \frac{1}{x}\,dx$

$x = 2 \rightarrow u = \ln(2)$

$x = 16 \rightarrow u = \ln(16) = 4\ln(2)$

48)

$$\int \frac{\sec(y)\tan(y)}{2+\sec(y)}\,dy = \int \frac{1}{u}\,du = \ln(|u|) + C =$$

$$= \ln(|2+\sec(y)|) + C$$

$u = 2 + \sec(y)$

$du = \sec(y)\tan(y)\,dy$