7.3)

2) a) $e^{-0.01t} = 1000$

$-0.01t = \ln(1000)$

$t = \frac{-3\ln(10)}{0.01}$

b) $e^{kt} = \frac{1}{10}$

$kt = -\ln(10)$

$t = \frac{-\ln(10)}{k}$

c) $e^{\ln(2)t} = \frac{1}{2}$

$2^t = \frac{1}{2}$

$t = \log_2\left(\frac{1}{2}\right)$

$t = -1$

10) $y = (1+2x)e^{-2x}$

$y' = 2e^{-2x} + (1+2x)e^{-2x} \cdot -2$

18) $y = \ln(2e^{-t}\sin(t)) = \ln(2) - \ln(e^{t}) + \ln(\sin(t))$

$y' = 1 + \frac{1}{\sin(t)} \cdot \cos(t) = 1 + \cot(t)$

22) $y = e^{\sin(t)}(\ln(t^2)+1)$

$y' = e^{\sin(t)}\cos(t)(2\ln(t)+1) + e^{\sin(t)} \cdot \left(\frac{2}{t}\right)$

26) $\frac{d \ln xy}{dx} = \frac{d e^{x+y}}{dx}$ $\quad \frac{d\, \ln(x)+\ln(y)}{dx} = \frac{d e^{x+y}}{dx}$

$\frac{1}{x} + \frac{1}{y}\frac{dy}{dx} = e^{x+y} \cdot \left(1 + \frac{dy}{dx}\right)$

$\frac{1}{x} - e^{x+y} = \frac{dy}{dx}\left(e^{x+y} - \frac{1}{y}\right)$

$\frac{dy}{dx} = \frac{e^{x+y} - \frac{1}{x}}{\frac{1}{y} - e^{x+y}}$

34) $\int 2e^{2x-1}\,dx = \int e^u\,du = e^u + c = e^{2x-1} + c$

$u = 2x-1$

$du = 2\,dx$

36) $\int_0^{\ln(16)} e^{\frac{x}{4}}\,dx = 4\int_0^2 e^u\,du = 4\left[e^u\right]_0^2 = 4(e^2 - e^0) =$

$= 4e^2 - 4$

$u = \frac{x}{4}$

$du = \frac{1}{4}\,dx$

$x = 0 \to u = 0$

$x = \ln(16) \to u = \frac{\ln(16)}{4} = \ln(2)$

40) $\int t^3 e^{(t^4)}\,dt = \frac{1}{4}\int e^u\,du = \frac{1}{4}e^u + c = \frac{1}{4}e^{(t^4)} + c$

$u = t^4$

$du = 4t^3\,dt$

48) $\int_0^{\sqrt{\ln(\pi)}} 2xe^{x^2}\cos(e^{x^2})\,dx = \int_1^{\pi}\cos(u)\,du = \left[\sin(u)\right]_1^{\pi} =$

$= \sin(\pi) - \sin(1) =$

$= \underline{\underline{-\sin(1)}}$

$u = e^{x^2}$

$du = 2xe^{x^2}\,dx$

$x = 0 \Rightarrow u = 1$

$x = \sqrt{\ln(\pi)} \Rightarrow u = \pi$

50) $\int \frac{dx}{1+e^x} = \int \underbrace{\frac{e^x}{(1+e^x)^2}\,dx}_{du} \cdot \frac{(1+e^x)}{e^x} = \int \frac{1}{u}\,du =$

$= \ln(|u|) + c =$

$= \underline{\ln\left(\frac{e^x}{1+e^x}\right) + c}$

$u = \frac{e^x}{1+e^x}$

$du = \frac{e^x(1+e^x) - e^x \cdot e^x}{(1+e^x)^2}\,dx = \frac{e^x}{(1+e^x)^2}\,dx$

56) $y = 3^{-x} = e^{-\ln(3)x}$

$y' = 3^{-x} \cdot (-\ln(3))$

58) $y = 2^{(s^2)} = e^{\ln(2)(s^2)}$

$y' = 2^{(s^2)} \ln(2) 2s$

68) $y = \log_3(1+\theta \ln(3)) = \dfrac{\ln(1+\theta\ln(3))}{\ln(3)}$

$y' = \dfrac{1}{1+\theta\ln(3)} \cdot \dfrac{\ln(3)}{\ln(3)} = \dfrac{1}{1+\theta\ln(3)}$

70) $y = \log_{25}(e^x) - \log_5(\sqrt{x}) = x\log_{25}(e) - \frac{1}{2}\log_5(x) =$

$= x\log_{25}(e) - \frac{1}{2}\dfrac{\ln(x)}{\ln(5)}$

$y' = \log_{25}(e) - \dfrac{1}{2\ln(5)x}$

72) $y = \log_3(r) \cdot \log_9(r) = \dfrac{\ln(r)}{\ln(3)} \cdot \dfrac{\ln(r)}{\ln(9)}$

$y' = \dfrac{1}{r\ln(3)} \cdot \log_9(r) + \log_3(r) \cdot \dfrac{1}{r\ln(9)}$

78) $y = \dfrac{\theta 5^\theta}{2-\log_5(\theta)}$ $\quad y' = \dfrac{(5^\theta + \theta\ln(5)5^\theta) - \theta 5^\theta \frac{1}{\theta\ln(5)}}{(2-\log_5(\theta))^2}$

92) $\int \frac{x\,2^{x^2}}{1+2^{x^2}}\,dx = \frac{1}{\ln(2)\,2}\int \frac{1}{u}\,du = \frac{\ln(|u|)}{\ln(2)\cdot 2} + c =$

$= \frac{\ln|1+2^{x^2}|}{\ln(2)\cdot 2} + c = \log_4(1+2^{x^2}) + c$

$u = 1+2^{x^2}$

$du = 2^{x^2}\cdot\ln(2)\cdot 2x\,dx$

98) $\int_1^4 \frac{\log_2(x)}{x}\,dx = \ln(2)\int_0^2 u\,du = \ln(2)\left[\frac{u^2}{2}\right]_0^2 =$

$= \ln(2)\,[2] = \underline{\underline{\ln(4)}}$ //

$u = \log_2(x)$

$du = \frac{1}{\ln(2)\,x}\,dx$

$x=4,\ u = 2$

$x=1 \rightarrow u = 0$

106) $\int \frac{dx}{x(\log_8 x)^2} = \ln(8)\int \frac{1}{u^2}\,du = -\ln(8)\,u^{-1} + c =$

$= \frac{-\ln(8)}{\log_8(x)} + c$

$u = \log_8 x$

$du = \frac{1}{\ln(8)\cdot x}\,dx$

112) $y = x^2 + x^{2x}$

$y_1 = x^2 \qquad y_1' = 2x$

$y_2 = x^{2x} \qquad \ln(y_2) = 2x\ln(x)$

$\frac{1}{y_2}\frac{dy_2}{dx} = 2\ln(x) + 2\frac{x}{x}$

$\frac{dy_2}{dx} = y_2\,2(\ln(x)+1)$

$\underline{\underline{y' = 2x + x^{2x}\cdot 2(\ln(x)+1)}}$ //

114) $y = t^{\sqrt{t}}$

$\ln(y) = \sqrt{t}\ \ln(t)$

$\frac{dy}{dt} = y \cdot \left(\frac{\ln(t)}{2\sqrt{t}} + \frac{\sqrt{t}}{t} \right) =$

$= t^{\sqrt{t}} \cdot \left(\frac{\ln(t) + 2}{2\sqrt{t}} \right)$

116) $y = x^{\sin(x)}$ $\ln(y) = \sin(x) \ln(x)$

$\frac{dy}{dx} = y \cdot \left(\cos(x) \ln(x) + \frac{\sin(x)}{x} \right)$

$\frac{dy}{dx} = x^{\sin(x)} \left(\cos(x) \ln(x) + \frac{\sin(x)}{x} \right)$

118) $y = (\ln(x))^{\ln(x)}$ $\ln(y) = \ln(x) \ln(\ln(x))$

$\frac{dy}{dx} = y \cdot \left(\frac{\ln(\ln(x))}{x} + \frac{\ln(x)}{\ln(x)} \cdot \frac{1}{x} \right)$

$= (\ln(x))^{\ln(x)} \cdot \left(\frac{\ln(\ln(x))}{x} + \frac{1}{x} \right)$

120) $y = 2e^{\sin(\frac{x}{2})}$

$y' = 2e^{\sin(\frac{x}{2})} \cdot \cos(\frac{x}{2}) \cdot \frac{1}{2} = e^{\sin(\frac{x}{2})} \cdot \cos(\frac{x}{2})$

$e^{\sin(\frac{x}{2})} \cos(\frac{x}{2}) = 0 \iff \cos(\frac{x}{2}) = 0$

$\frac{x}{2} = \frac{\pi}{2} + k\pi$

$x = \pi + 2k\pi$.

$y'' = \frac{e}{2}^{\sin(\frac{x}{2})} \cos^2(\frac{x}{2}) + (-\sin(\frac{x}{2})) e^{\sin(\frac{x}{2})}$ at $\pi + 2k\pi$ $\frac{e}{2}^{\sin(\frac{x}{2})} \cos^2(\frac{x}{2}) = 0$

and the sign of the second term depends only on $-\sin(\frac{x}{2})$. This is negative for $\pi + 4k\pi$ and positive for $3\pi + 4k\pi$

thus we get local and global maxima at $\pi + 4k\pi$ and local and global minima at $3\pi + 4k\pi$.

The Maxima are e and the minima are e^{-1}.

7.5)

2) $$\lim_{x\to 0} \frac{\sin(5x)}{x} = \text{"}\frac{0}{0}\text{"}$$

$$\overset{L'H.}{=} \lim_{x\to 0} \frac{5\cos(5x)}{1} = 5$$

6) $$\lim_{x\to\infty} \frac{2x^2+3x}{x^3+x+1} = \text{"}\frac{\infty}{\infty}\text{"}$$

$$\overset{L'H.}{=} \lim_{x\to\infty} \frac{4x+3}{3x^2+1} = \text{"}\frac{\infty}{\infty}\text{"}$$

$$\overset{L'H}{=} \lim_{x\to\infty} \frac{4}{6x+1} = 0$$

18) $$\lim_{\theta\to -\frac{\pi}{3}} \frac{3\theta+\pi}{\sin(\theta+\frac{\pi}{3})} = \text{"}\frac{0}{0}\text{"}$$

$$\overset{L'H}{=} \lim_{\theta\to -\frac{\pi}{3}} \frac{3}{\cos(\theta+\frac{\pi}{3})} = \frac{3}{1} = 3$$

24) $$\lim_{t\to 0} \frac{t\sin(t)}{1-\cos(t)} = \text{"}\frac{0}{0}\text{"}$$

$$\overset{L'H.}{=} \lim_{t\to 0} \frac{\sin(t)+t\cos(t)}{\sin(t)} = \text{"}\frac{0}{0}\text{"}$$

$$\overset{L'H.}{=} \lim_{t\to 0} \frac{\cos(t)-t\sin(t)+\cos(t)}{\cos(t)} = \frac{1+0+1}{1} = 2$$

28) $$\lim_{\theta\to 0} \frac{(\frac{1}{2})^{\theta}-1}{\theta} = \text{"}\frac{0}{0}\text{"}$$

$$\overset{L'H}{=} \lim_{\theta\to 0} \frac{(\frac{1}{2})^{\theta}\ln(\frac{1}{2})}{1} = \ln(\frac{1}{2})$$

40) $\lim_{x\to 0^+} \left(\csc(x) - \cot(x) + \cos(x) \right) =$

$$= \lim_{x\to 0^+} \left(\frac{1}{\sin(x)} - \frac{\cos(x)}{\sin(x)} + \frac{\cos^2(x)}{\cos(x)} \right) =$$

$$= \lim_{x\to 0^+} \left(\frac{1-\cos(x)}{\sin(x)} + \cos(x) \right) =$$

$$= \lim_{x\to 0^+} \left(\frac{1-\cos(x)}{\sin(x)} \right) + \lim_{x\to 0^+} \cos(x) = \text{"}\frac{0}{0}\text{"} + 1$$

$$\overset{L'H}{=} \lim_{x\to 0^+} \left(\frac{+\sin(x)}{\cos(x)} \right) + 1 = 0 + 1 = \underline{\underline{1}}$$

46) $\lim_{x\to\infty} x^2 e^{-x} = \lim_{x\to\infty} \frac{x^2}{e^x} = \text{"}\frac{\infty}{\infty}\text{"}$

$$\overset{L'H}{=} \lim_{x\to\infty} \frac{2x}{e^x} = \text{"}\frac{\infty}{\infty}\text{"} = \cancel{\lim_{x\to\infty}}$$

$$\overset{L'H}{=} \lim_{x\to\infty} \frac{2}{e^x} = \underline{\underline{0}}$$

50) $\lim_{x\to 0} \frac{\sin(3x) - 3x + x^2}{\sin(x)\sin(2x)} = \text{"}\frac{0}{0}\text{"}$

$$\overset{L'H}{=} \lim_{x\to 0} \frac{3\cos(3x) - 3 + 2x}{\cos(x)\sin(2x) + 2\cos(2x)\sin(x)} = \text{"}\frac{0}{0}\text{"}$$

$$\overset{L'H}{=} \lim_{x\to 0} \frac{-9\sin(3x) + 2}{-\sin(x)\sin(2x) + 2\cos(2x)\cos(x) - 4\sin(2x)\sin(x) + 2\cos(2x)\cos(x)} = \frac{2}{4} = \underline{\underline{\frac{1}{2}}}$$

54)

$$\lim_{x\to e^+} \ln(x)^{\frac{1}{x-e}} = "1^{\infty}"$$

$$= e^{\lim_{x\to e^+} \frac{\ln(\ln(x))}{x-e}}$$

$$\lim_{x\to e^+} \frac{\ln(\ln(x))}{x-e} = "\frac{0}{0}"$$

$$\overset{L'H}{=} \lim_{x\to e^+} \frac{\frac{1}{\ln(x)} \cdot \frac{1}{x}}{1} = \frac{1}{1} \cdot \frac{1}{e}$$

So $\lim_{x\to e^+} \ln(x)^{\frac{1}{x-e}} = \underline{\underline{e^{\frac{1}{e}}}}$ //

58)

$$\lim_{x\to 0} (e^x+x)^{\frac{1}{x}} = e^{\lim_{x\to 0} \frac{\ln(e^x+x)}{x}}$$

$$\lim_{x\to 0} \frac{\ln(e^x+x)}{x} = "\frac{0}{0}"$$

$$\overset{L'H.}{=} \lim_{x\to 0} \frac{\frac{1}{e^x+x} \cdot (e^x+1)}{1} = \frac{1+1}{1+0} = 2$$

So $\underline{\underline{\lim_{x\to 0} (e^x+x)^{\frac{1}{x}} = e^2}}$ //

60) $\lim_{x\to 0^+} \left(1+\frac{1}{x}\right)^x = "1^0"$

$\lim_{x\to 0^+} e^{\ln(1+\frac{1}{x})\, x} = e^{\lim_{x\to 0^+} \ln(1+\frac{1}{x})\cdot x}$

$\lim_{x\to 0^+} \frac{\ln(1+\frac{1}{x})}{\frac{1}{x}} = "\frac{0}{0}"$

$\overset{L'H}{=} \lim_{x\to 0^+} \frac{\frac{1}{1+\frac{1}{x}} \cdot \frac{-1}{x^2}}{\frac{-1}{x^2}} = \lim_{x\to 0^+} \frac{1}{1+\frac{1}{x}} = \frac{1}{1+0} = \underline{\underline{1}}//$

so $\lim_{x\to 0^+} \left(1+\frac{1}{x}\right)^x = e^1 = \underline{\underline{e}}//$

66) $\lim_{x\to 0^+} \sin(x)\cdot \ln(x) = \lim_{x\to 0^+} \frac{\ln(x)}{\csc(x)} = "\frac{-\infty}{\infty}"$

$\overset{L'H}{=} \lim_{x\to 0^+} \frac{\frac{1}{x}}{-\csc(x)\cot(x)} = \lim_{x\to 0^+} \frac{-\sin(x)\tan(x)}{x} = "\frac{0}{0}"$

$\overset{L'H}{=} \lim_{x\to 0^+} -\frac{\cos(x)\tan(x) - \sin(x)\sec^2(x)}{1} =$

$= \frac{0}{1} = \underline{\underline{0}}//$

68)

$$\lim_{x\to 0^+} \frac{\sqrt{x}}{\sqrt{\sin(x)}} = \lim_{x\to 0^+} \sqrt{\frac{x}{\sin(x)}} =$$

$$= \sqrt{\lim_{x\to 0^+} \frac{x}{\sin(x)}} = \sqrt{\lim_{x\to 0} \frac{1}{\frac{\sin(x)}{x}}} = \sqrt{\frac{1}{\lim_{x\to 0^+} \frac{\sin(x)}{x}}} = \sqrt{\frac{1}{1}} = \underline{\underline{1}}$$

72)

$$\lim_{x\to -\infty} \frac{2^x + 4^x}{5^x - 2^x} = \lim_{x\to -\infty} \frac{2^x}{2^x} \frac{1 + \frac{4^x}{2^x}}{\frac{5^x}{2^x} - 1} =$$

$$= \lim_{x\to -\infty} \frac{1 + (2)^x}{\left(\frac{5}{2}\right)^x - 1} = \frac{1+0}{0-1} = -1$$

74)

$$\lim_{x\to 0^+} \frac{x}{e^{-\frac{1}{x}}} = \lim_{x\to 0^+} \frac{e^{\frac{1}{x}}}{\frac{1}{x}} = \text{“}\frac{\infty}{\infty}\text{”}$$

$$\overset{L'H}{=} \lim_{x\to 0^+} \frac{e^{\frac{1}{x}} \cdot -\frac{1}{x^2}}{-\frac{1}{x^2}} = \lim_{x\to 0^+} e^{\frac{1}{x}} = \infty.$$

76) a) is incorrect since the limit at the second step is no longer of type “$\frac{0}{0}$” or “$\frac{\infty}{\infty}$” and so L'Hôpital's rule does not apply.