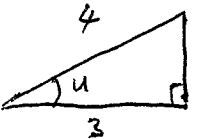
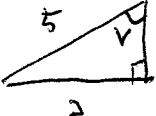


P 5.7)

2) $\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$ since the domain which we restrict $\sin(x)$ to so we could define $\sin^{-1}(x)$ was $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and $\frac{\pi}{6}$ is the only angle in this domain such that $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$.

4) $\tan^{-1}(-\sqrt{3}) = -\frac{\pi}{3}$

6)  $u = \cos^{-1}\left(\frac{3}{4}\right) \approx 41.41^\circ \approx \underline{\underline{0.723}}$

8)  $v = \sin^{-1}\left(\frac{3}{5}\right) \approx 36.87^\circ \approx \underline{\underline{0.644}}$

16) $e^{\tan(t)} = 500$
 $\tan(t) = \ln(500)$
 $t = \tan^{-1}(\ln(500)) \approx 80.86^\circ \approx \underline{\underline{1.41}}$

18) $\sin(\tan(y)) = 0.6$
 $\tan(y) = \sin^{-1}(0.6)$
 $y = \tan^{-1}(\sin^{-1}(0.6)) \approx \underline{\underline{0.572}}$

1. The first part of the problem is to find the value of $\int_0^1 x^2 dx$

using the definition of the definite integral.

We start by dividing the interval $[0, 1]$ into n subintervals.

$$\Delta x = \frac{1-0}{n} = \frac{1}{n}$$

$$x_k = \frac{k}{n}$$

$$\int_0^1 x^2 dx = \lim_{n \rightarrow \infty} \sum_{k=1}^n \left(\frac{k}{n}\right)^2 \frac{1}{n}$$



$$= \lim_{n \rightarrow \infty} \frac{1}{n^3} \sum_{k=1}^n k^2$$

$$= \lim_{n \rightarrow \infty} \frac{1}{n^3} \left(\frac{n(n+1)(2n+1)}{6} \right)$$

$$= \lim_{n \rightarrow \infty} \frac{(n+1)(2n+1)}{6n^2}$$

$$= \frac{1 \cdot 2}{6} = \frac{1}{3}$$

Therefore, the value of the integral is $\frac{1}{3}$.

$$\int_0^1 x^2 dx = \frac{1}{3}$$

Q.E.D.

$$\int_0^1 x^2 dx = \frac{1}{3}$$

P. 5.8)
2)

$$\sin^{-1}(t) = -\frac{2\pi}{7}$$

$$a) \sin^{-1}(-t) = -\sin^{-1}(t) = \frac{2\pi}{7}$$

$$b) \cos^{-1}(-t) = \pi - \cos^{-1}(t) = \pi - \left(\frac{\pi}{2} - \sin^{-1}(t)\right) = \\ = \frac{\pi}{2} + \sin^{-1}(t) = \frac{7\pi}{14} - \frac{4\pi}{14} = \frac{3\pi}{14}$$

$$b) \cos^{-1}(t) = \frac{\pi}{2} - \sin^{-1}(t) = \frac{\pi}{2} + \frac{2\pi}{7} = \frac{11\pi}{14}$$

$$4) \tan^{-1}(t) = -\frac{4\pi}{11} \quad \text{so} \quad t < 0$$

$$\text{thus } a) \tan^{-1}\left(\frac{1}{t}\right) = -\frac{\pi}{2} - \tan^{-1}(t) = -\frac{\pi}{2} + \frac{4\pi}{11} = -\frac{3\pi}{22}$$

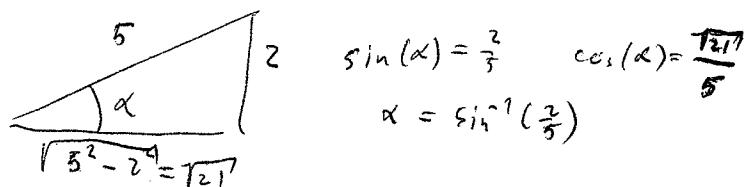
$$b) \tan^{-1}(-t) = -\tan^{-1}(t) = \frac{4\pi}{11}$$

$$c) \tan^{-1}\left(-\frac{1}{t}\right) = -\tan^{-1}\left(\frac{1}{t}\right) = \frac{3\pi}{22}$$

$$6) \tan(\tan^{-1}(5)) = 5$$

$$8) \cos^{-1}\left(\cos\left(\frac{1}{2}\right)\right) = \frac{1}{2}$$

$$16) \cos\left(\sin^{-1}\left(\frac{2}{5}\right)\right) = \frac{\sqrt{21}}{5}$$



18)

use picture from 16)

$$\tan\left(\sin^{-1}\left(\frac{2}{5}\right)\right) = \frac{2}{\sqrt{21}}$$

1. $\frac{1}{x^2} = x^{-2}$
 $\frac{d}{dx} x^{-2} = -2x^{-3} = -\frac{2}{x^3}$

2. $\frac{1}{x^3} = x^{-3}$
 $\frac{d}{dx} x^{-3} = -3x^{-4} = -\frac{3}{x^4}$

3. $\frac{1}{x^4} = x^{-4}$
 $\frac{d}{dx} x^{-4} = -4x^{-5} = -\frac{4}{x^5}$

4. $\frac{1}{x^5} = x^{-5}$
 $\frac{d}{dx} x^{-5} = -5x^{-6} = -\frac{5}{x^6}$

5. $\frac{1}{x^6} = x^{-6}$
 $\frac{d}{dx} x^{-6} = -6x^{-7} = -\frac{6}{x^7}$

6. $\frac{1}{x^7} = x^{-7}$
 $\frac{d}{dx} x^{-7} = -7x^{-8} = -\frac{7}{x^8}$

7.6)

$$14) \lim_{x \rightarrow -1^+} \cos^{-1}(x) = \underline{\underline{\pi}} \quad 18) \lim_{x \rightarrow -\infty} \sec^{-1}(x) = \underline{\underline{\frac{\pi}{2}}}$$

$$22) y' = \left(\cos^{-1}\left(\frac{1}{x}\right) \right)' = \frac{-1}{\sqrt{1-x^{-2}}} \cdot -x^{-2} = \underline{\underline{\frac{1}{\sqrt{1-\frac{1}{x^2}} \cdot x^2}}}$$

$$26) y' = \left(\sec^{-1}(5x) \right)' = \frac{1}{5x \sqrt{(5x)^2 - 1}} \cdot 5$$

$$30) y' = \left(\sin^{-1}\left(\frac{3}{e^2}\right) \right)' = \frac{-1}{\sqrt{1-\left(\frac{3}{e^2}\right)^2}} \cdot 3 \cdot e^{-3} = \underline{\underline{\frac{9e^{-3}}{\sqrt{1-9e^{-4}}}}}$$

$$44) \int \frac{dx}{\sqrt{1-4x^2}} = \int \frac{dx}{\sqrt{1-(2x)^2}} = \frac{1}{2} \int \frac{du}{\sqrt{1-u^2}} =$$

$$u = 2x \\ du = 2 dx$$

$$= \frac{\sin^{-1}(u)}{2} + C = \frac{\sin^{-1}(2x)}{2} + C = \underline{\underline{\frac{\sin^{-1}(2x)}{2} + C}}$$

$$48) \int \frac{dx}{x \sqrt{5x^2-4}} = \int \frac{dx}{x \sqrt{\frac{5x^2}{4}-1} \cdot \frac{1}{2}} = \int \frac{dx}{x \cdot \frac{1}{2} \cdot \sqrt{\left(\frac{\sqrt{5}}{2}x\right)^2-1}} =$$

$$u = \frac{\sqrt{5}}{2}x \quad x = \frac{2u}{\sqrt{5}}$$

$$du = \frac{\sqrt{5}}{2} dx$$

$$= \frac{2}{\sqrt{5}} \cdot \frac{1}{\frac{1}{2}} \cdot \int \frac{\frac{2u}{\sqrt{5}} du}{u \sqrt{u^2-1}} =$$

$$= \frac{1}{2} \int \frac{du}{u \sqrt{u^2-1}} = \frac{1}{2} \sec^{-1}(|u|) + C = \underline{\underline{\frac{1}{2} \sec^{-1}\left(\left|\frac{\sqrt{5}}{2}x\right|\right) + C}}$$

$$54) \int_{-\frac{2}{3}}^{-\frac{\sqrt{2}}{3}} \frac{dy}{y\sqrt{9y^2-1}} = \int_{-\frac{2}{3}}^{-\frac{\sqrt{2}}{3}} \frac{dy}{y\sqrt{(3y)^2-1}} = \frac{1}{3} \int_{-2}^{-\sqrt{2}} \frac{du}{\frac{u}{3}\sqrt{u^2-1}} =$$

$$u = 3y \quad y = \frac{u}{3}$$

$$du = 3dy$$

$$x = -\frac{2}{3} \Rightarrow u = -2$$

$$y = -\frac{\sqrt{2}}{3} \Rightarrow u = -\sqrt{2}$$

$$= \int_{-2}^{-\sqrt{2}} \frac{du}{u\sqrt{u^2-1}} = \left[\sec^{-1}(|1-\sqrt{2}|) - \sec^{-1}(|1-2|) \right] =$$

$$= \frac{\pi}{4} - \frac{\pi}{3} = \underline{\underline{-\frac{\pi}{12}}}$$

$$58) \int \frac{dx}{1+(3x+1)^2} = \frac{1}{3} \int \frac{1}{1+u^2} du = \frac{\tan^{-1}(u)}{3} + C = \frac{\tan^{-1}(3x+1)}{3} + C$$

$$u = 3x+1$$

$$du = 3dx$$

$$64) \int_1^{e^{\frac{\pi}{4}}} \frac{4dt}{1+(1+\ln^2(t))} = \int_0^{\frac{\pi}{4}} \frac{4dy}{1+u^2} = 4\tan^{-1}\left(\frac{\pi}{4}\right) - 4\tan^{-1}(0) =$$

$$= \underline{\underline{4\tan^{-1}\left(\frac{\pi}{4}\right)}}$$

$$u = \ln(t)$$

$$du = \frac{1}{t} dt$$

$$t=1 \rightarrow u=0$$

$$t=e^{\frac{\pi}{4}} \rightarrow u = \frac{\pi}{4}$$

$$72) \int \frac{dy}{y^2+6y+10} = \int \frac{dy}{(y+3)^2+1} = \int \frac{1}{u^2+1} = \tan^{-1}(u) + C =$$

$$= \underline{\underline{\tan^{-1}(y+3) + C}}$$

$$u = y+3$$

$$du = dy$$

78)

$$\int \frac{t^3 - 2t^2 + 3t - 4}{t^2 + 1} dt$$

We do long division to simplify the problem.

$$\begin{array}{r} t - 2 \\ (t^2 + 1) \overline{) t^3 - 2t^2 + 3t - 4} \\ \underline{-(t^3 \quad + t)} \\ 0 - 2t^2 + 2t - 4 \\ \underline{-(-2t^2 \quad - 2)} \\ 0 \quad 2t - 2 \end{array}$$

$$\text{so } t^3 - 2t^2 + 3t - 4 = (t - 2)(t^2 + 1) + 2t - 2$$

$$\int \frac{(t - 2)(t^2 + 1) + 2t - 2}{t^2 + 1} dt = \int t - 2 + \frac{2t}{t^2 + 1} - \frac{2}{t^2 + 1} dt =$$

$$= \int t - 2 dt + \int \frac{2t}{t^2 + 1} dt - \int \frac{2}{t^2 + 1} dt =$$

$$= \frac{t^2}{2} - 2t + 2 \ln(|t^2 + 1|) - 2 \tan^{-1}(t^2 + 1) + C //$$

$$84) \int \frac{\sqrt{\tan^{-1}(x)}}{1 + x^2} dx = \int \sqrt{u} du = \frac{2}{3} u^{\frac{3}{2}} + C = \frac{2}{3} (\tan^{-1}(u))^{\frac{3}{2}} + C //$$

$$u = \tan^{-1}(x)$$

$$du = \frac{1}{1 + x^2} dx$$

94)

$$\lim_{x \rightarrow 0} \frac{2 \tan^{-1}(3x^2)}{7x^2} = \frac{0}{0}$$

L'H

$$\lim_{x \rightarrow 0} \frac{2 \frac{1}{1+(3x^2)^2} \cdot 6x}{7 \cdot 2x} = \lim_{x \rightarrow 0} \frac{6}{7(1+(3x^2)^2)} = \frac{6}{7}$$