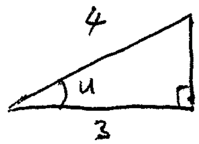
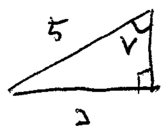
P 5.7)

2) $\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$ since the domain which we restricte $\sin(x)$ to so we could define $\sin^{-1}(x)$ was $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ and $\frac{\pi}{6}$ is the only angle in this domain such that $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$.

4) $\tan^{-1}(-\sqrt{3}) = -\frac{\pi}{3}$

6) 

$u = \cos^{-1}\left(\frac{3}{4}\right) \cong 41.41° \cong 0.723$

8) 

$v = \sin^{-1}\left(\frac{3}{5}\right) \cong 36.87° \cong 0.644$

16)
$$e^{\tan(t)} = 500$$
$$\tan(t) = \ln(500)$$
$$t = \tan^{-1}(\ln(500)) \cong 80.86° \cong 1.41$$

18)
$$\sin(\tan(y)) = 0.6$$
$$\tan(y) = \sin^{-1}(0.6)$$
$$y = \tan^{-1}(\sin^{-1}(0.6)) \cong 0.572$$

P. 5.8)

2) $\sin^{-1}(t) = -\frac{2\pi}{7}$

a) $\sin^{-1}(-t) = -\sin^{-1}(t) = \frac{2\pi}{7}$

c) $\cos^{-1}(-t) = \pi - \cos^{-1}(t) = \pi - (\frac{\pi}{2} - \sin^{-1}(t)) =$

$= \frac{\pi}{2} + \sin^{-1}(t) = \frac{7\pi}{14} - \frac{4\pi}{14} = \frac{3\pi}{14}$

b) $\cos^{-1}(t) = \frac{\pi}{2} - \sin^{-1}(t) = \frac{\pi}{2} + \frac{2\pi}{7} = \frac{11\pi}{14}$

4) $\tan^{-1}(t) = -\frac{4\pi}{11}$ so $t < 0$

thus a) $\tan^{-1}(\frac{1}{t}) = -\frac{\pi}{2} - \tan^{-1}(t) = -\frac{\pi}{2} + \frac{4\pi}{11} = -\frac{3\pi}{22}$

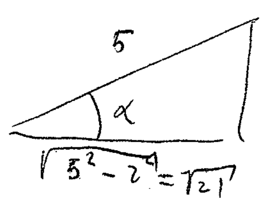
b) $\tan^{-1}(-t) = -\tan^{-1}t) = \frac{4\pi}{11}$

c) $\tan^{-1}(-\frac{1}{t}) = -\tan^{-1}(\frac{1}{t}) = \frac{3\pi}{22}$

6) $\tan(\tan^{-1}(5)) = 5$

8) $\cos^{-1}(\cos(\frac{1}{2})) = \frac{1}{2}$

16) $\cos(\sin^{-1}(\frac{2}{5})) = \frac{\sqrt{21}}{5}$



$\sin(\alpha) = \frac{2}{5}$ $\cos(\alpha) = \frac{\sqrt{21}}{5}$

$\alpha = \sin^{-1}(\frac{2}{5})$

18) use picture from 16)

$\tan(\sin^{-1}(\frac{2}{5})) = \frac{2}{\sqrt{21}}$

7.6) 14) $\lim_{x \to -1^+} \cos^{-1}(x) = \pi$ 18) $\lim_{x \to -\infty} \sec^{-1}(x) = \frac{\pi}{2}$

22) $y' = \left(\cos^{-1}\left(\frac{1}{x}\right)\right)' = \frac{-1}{\sqrt{1-x^{-2}}} \cdot -x^{-2} = \frac{1}{\sqrt{1-\frac{1}{x^2}} \cdot x^2}$

26) $y' = (\sec^{-1}(5s))' = \frac{1}{|5s|\sqrt{(5s)^2-1}} \cdot 5$

30) $y' = \left(\sin^{-1}\left(\frac{3}{t^2}\right)\right)' = \frac{-1}{\sqrt{1-\left(\frac{3}{t^2}\right)^2}} \cdot 3 \cdot 3t^{-3} = \frac{9t^{-3}}{\sqrt{1-9t^{-4}}}$

44) $\int \frac{dx}{\sqrt{1-4x^2}} = \int \frac{dx}{\sqrt{1-(2x)^2}} = \frac{1}{2}\int \frac{du}{\sqrt{1-u^2}} =$

$u = 2x$
$du = 2dx$

$= \frac{\sin^{-1}(u)}{2} + c = \frac{\sin^{-1}(u)}{2} + c = \frac{\sin^{-1}(2x)}{2} + c$

48) $\int \frac{dx}{x\sqrt{5x^2-4}} = \int \frac{dx}{x\sqrt{\frac{5x^2}{4}-1} \cdot \sqrt{4}} = \int \frac{dx}{x \cdot \sqrt{4} \cdot \sqrt{\left(\frac{\sqrt{5}x}{2}\right)^2-1}} =$

$u = \frac{\sqrt{5}x}{2}$ $x = \frac{2u}{\sqrt{5}}$
$du = \frac{\sqrt{5}}{2}dx$

$= \frac{2}{\sqrt{5}} \cdot \frac{1}{\sqrt{4}} \cdot \int \frac{du}{\frac{2u}{\sqrt{5}}\sqrt{u^2-1}} =$

$= \frac{1}{2}\int \frac{du}{u\sqrt{u^2-1}} = \frac{1}{2}\sec^{-1}(|u|) + c = \frac{1}{2}\sec^{-1}\left(\left|\frac{\sqrt{5}}{2}x\right|\right) + c$

54) $$\int_{-\frac{2}{3}}^{-\frac{\sqrt{2}}{3}} \frac{dy}{y\sqrt{9y^2-1}} = \int_{-\frac{2}{3}}^{-\frac{\sqrt{2}}{3}} \frac{dy}{y\sqrt{(3y)^2-1}} = \frac{1}{3}\int_{-2}^{-\sqrt{2}} \frac{du}{\frac{u}{3}\sqrt{u^2-1}} =$$

$u = 3y \qquad y = \frac{u}{3}$

$du = 3\,dy$

$\cancel{x}y = -\frac{2}{3} \Rightarrow u = -2$

$y = -\frac{\sqrt{2}}{3} \quad u = -\sqrt{2}$

$$= \int_{-2}^{-\sqrt{2}} \frac{du}{u\sqrt{u^2-1}} = [\sec^{-1}(|-\sqrt{2}|) - \sec^{-1}(|-2|)) =$$

$$= \frac{\pi}{4} - \frac{\pi}{3} = -\frac{\pi}{12}$$

58) $$\int \frac{dx}{1+(3x+1)^2} = \frac{1}{3}\int \frac{1}{1+u^2}\,du = \frac{\tan^{-1}(u)}{3} + c = \frac{\tan^{-1}(3x+1)}{3} + c$$

$u = 3x+1$

$du = 3\,dx$

64) $$\int_1^{e^{\frac{\pi}{4}}} \frac{4\,dt}{t(1+\ln^2(t))} = \int_0^{\frac{\pi}{4}} \frac{4\,du}{1+u^2} = 4\tan^{-1}\left(\frac{\pi}{4}\right) - 4\tan^{-1}(0) =$$

$$= \tan^{-1}\left(\frac{\pi}{4}\right)$$

$u = \ln(t)$

$du = \frac{1}{t}\,dt$

$\cancel{t}t = 1 \to u = 0$

$t = e^{\frac{\pi}{4}} \to u = \frac{\pi}{4}$

~~7~~2) $$\int \frac{dy}{y^2+6y+10} = \int \frac{dy}{(y+3)^2+1} = \int \frac{1}{u^2+1} = \tan^{-1}(u) + c =$$

$$= \tan^{-1}(y+3) + c$$

$u = y+3$

$du = dy$

78)

$$\int \frac{t^3-2t^2+3t-4}{t^2+1}\,dt$$

We do long division to simplify the problem.

$$\begin{array}{r l}
 & \quad t \; -2 \\
(t^2+1) & \overline{\big)\, t^3-2t^2+3t-4} \\
 & -(t^3 \qquad\quad + t) \\
\hline
 & 0 \; -2t^2 + 2t - 4 \\
 & -(-2t^2 \qquad -2) \\
\hline
 & \quad 0 \quad 2t - 2
\end{array}$$

so $\quad t^3-2t^2+3t-4 = (t-2)(t^2+1) + 2t - 2$

$$\int \frac{(t-2)(t^2+1)+2t-2}{t^2+1}\,dt = \int t-2+\frac{2t}{t^2+1}-\frac{2}{t^2+1}\,dt =$$

$$= \int t-2\,dt + \int \frac{2t}{t^2+1}\,dt - \int \frac{2}{t^2+1}\,dt =$$

$u = \cancel{tt}^2+1$

$du = 2t\,dt$

$$= \frac{t^2}{2} - 2t + 2\ln(|t^2+1|) - 2\tan^{-1}(t^2+1) + C$$

84) $$\int \frac{\sqrt{\tan^{-1}(x)}}{1+x^2}\,dx = \int \sqrt{u}\,du = \frac{2}{3}u^{\frac{3}{2}} + C = \frac{2}{3}(\tan^{-1}(u))^{\frac{3}{2}} + C$$

$u = \tan^{-1}(x)$

$du = \frac{1}{1+x^2}\,dx$

94)

$$\lim_{x \to 0} \frac{2\tan^{-1}(3x^2)}{7x^2} = \text{"}\frac{0}{0}\text{"}$$

L'H

$$\lim_{x \to 0} \frac{2\,\frac{1}{1+(3x^2)^2}\cdot 6x}{7\cdot 2x} = \lim_{x \to 0} \frac{6}{7(1+(3x^2)^2)} = \frac{6}{7}$$