§7.7

#8 $\cosh 3x - \sinh 3x$

$$= \frac{e^{3x}+e^{-3x}}{2} - \frac{e^{3x}-e^{-3x}}{2}$$

$$= \frac{(e^{3x}+e^{-3x})-(e^{3x}-e^{-3x})}{2} = \frac{2e^{-3x}}{2} = e^{-3x}$$

#12. $$\cosh^2 x - \sinh^2 x = \left(\frac{e^x+e^{-x}}{2}\right)^2 - \left(\frac{e^x-e^{-x}}{2}\right)^2$$

$$= \frac{(e^x)^2+2\cdot e^x\cdot e^{-x}+(e^{-x})^2}{4} - \frac{(e^x)^2-2\cdot e^x\cdot e^{-x}+(e^{-x})^2}{4}$$

$$= \frac{(e^{2x}+2+e^{-2x})-(e^{2x}-2+e^{-2x})}{4}$$

$$= \frac{4}{4} = 1$$

#14 $y = \frac{1}{2}\sinh(2x+1)$

$$y' = \frac{1}{2}\cosh(2x+1)\cdot(2x+1)'$$

$$= \cosh(2x+1).$$

#35 $y = \sinh^{-1}(\tan x)$.

$$y' = \frac{1}{\sqrt{1+\tan^2 x}}\cdot(\tan x)'$$

$$= \frac{1}{\sqrt{\sec^2 x}}\cdot\sec^2 x$$

$$= \frac{\sec^2 x}{|\sec x|} = |\sec x|$$

#68 (a) $\displaystyle\int_0^{\frac{1}{3}} \frac{6\,dx}{\sqrt{1+9x^2}}$ set $u = 3x.$

$du = 3\,dx$

$$= \int_0^1 \frac{6 \cdot \frac{1}{3}\,du}{\sqrt{1+u^2}} \qquad dx = \tfrac{1}{3}\,du$$

$$= 2\int_0^1 \frac{1}{\sqrt{1+u^2}}\,du = 2\sinh^{-1} u \Big|_0^1$$

$$= 2\sinh^{-1}(1) - \sinh^{-1}(0)$$

(b) $\sinh^{-1}(1)$: $\dfrac{e^x - e^{-x}}{2} = 1$ i.e. $e^x - e^{-x} = 2$

$\Rightarrow e^{2x} - 2e^x - 1 = 0$

Let $u = e^x$.

$u^2 - 2u - 1 = 0.$

$\Rightarrow u = \dfrac{2 \pm \sqrt{8}}{2} = 1 \pm \sqrt{2}.$ (choose positive)

$e^x = 1 + \sqrt{2}.$

$x = \ln(1+\sqrt{2}).$

$\sinh^{-1}(0)$: $\dfrac{e^x - e^{-x}}{2} = 0$, i.e. $e^x = e^{-x}$

$\Rightarrow e^{2x} = 1$

$\Rightarrow 2x = 0$ i.e. $x = 0$.

$$\int_0^{\frac{1}{3}} \frac{6\,dx}{\sqrt{1+9x^2}} = 2\sinh^{-1}(1) - \sinh^{-1}(0)$$

$$= \ln(1+\sqrt{2}) - 0$$

$$= \ln(1+\sqrt{2})$$

P.26

#8 $(5+6i)(2+7i) = 5\cdot 2 + 6i\cdot 2 + 5\cdot 7i + 6i\cdot 7i$

$= 10 + 12i + 35i - 42$

$= -32 + 47i.$

#18 $(5+\sqrt{6}i)^2 = (5+\sqrt{6}i)(5+\sqrt{6}i)$

$= 5\cdot 5 + \sqrt{6}i\cdot 5 + 5\cdot\sqrt{6}i + \sqrt{6}i\cdot\sqrt{6}i$

$= 25 + 5\sqrt{6}i + 5\sqrt{6}i - 6$

$= 19 + 10\sqrt{6}i.$

#22 $(4+3i)^3 = (4+3i)(4+3i)(4+3i)$

$= (16 + 24i - 9)(4+3i)$

$= (7+24i)(4+3i)$

$= 28 + 117i - 72$

$= -44 + 117i$

#26 $i^{1003} = \underbrace{(i^4)(i^4)\cdots(i^4)}_{250 \text{ times}}\, i^3$

$= \underbrace{1\cdot 1\cdot 1\cdot\cdots\cdot 1}_{250 \text{ times}}\,(-i)$

$= -i.$

#32 $\dfrac{5+6i}{2+3i} = \dfrac{(5+6i)(2-3i)}{(2+3i)(2-3i)} = \dfrac{28-3i}{13}.$

#38

#38 Let

$(a+bi)^2 = 21-20i$. with a, b real

$(a+bi)^2 = a^2-b^2+2abi$

Thus $\begin{cases} a^2-b^2=21 \\ 2ab=-20 \end{cases} \Rightarrow \begin{cases} a^2-b^2=21 \\ ab=-10 \end{cases}$

$\Rightarrow a=5,\ b=-2$

$a=-5,\ b=2$

Thus $5-2i$ or $-5+2i$

#40 Let $z+z'=5$.

$zz'=11$

i.e. $z'=5-z$

Write $z=a+bi$ with a, b real

$z'=5-z=5-(a+bi)=(5-a)-bi$

$11=zz'=(a+bi)((5-a)-bi)$

$=a(5-a)+b^2+[(5-a)b-ab]i$

Thus, $\begin{cases} a(5-a)+b^2=11 \\ (5-a)b-ab=0. \end{cases}$

i.e. $\begin{cases} -a^2+5a+b^2=11 \\ 5b-2ab=0 \end{cases}$

$\Rightarrow b(5-2a)=0$. i.e. $b=0$ or $a=\frac{5}{2}$.

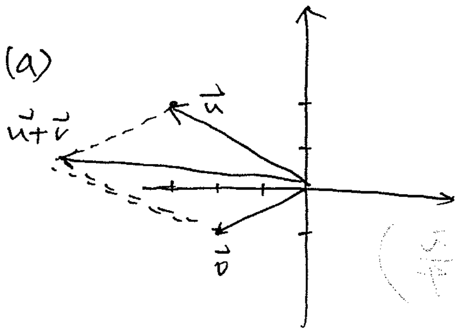
If $b=0 \Rightarrow a^2-5a+11=0$. $a=\frac{5\pm\sqrt{-19}}{2}$ (not real)

If $a=\frac{5}{2} \Rightarrow b^2=11+a^2-5a=\frac{19}{4}$. $b=\pm\frac{\sqrt{19}}{2}$

thus $z=\frac{5}{2}+\frac{\sqrt{19}}{2}$, $z'=\frac{5}{2}-\frac{\sqrt{19}}{2}$.

P6.7

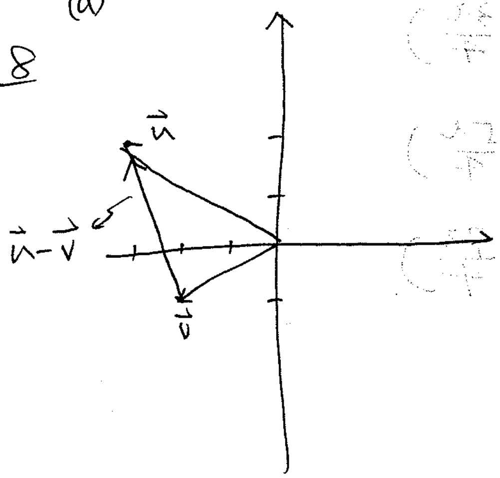
#6 (a)



(b) $\vec{u}+\vec{v} = (-3,2)+(-2,-1)$

$= (-5,1)$

#8 (a)



(b) $\vec{u}-\vec{v} = (-3,2)-(-2,-1)$

$= (-1,3)$

#12

$(3,-5)\cdot(-4,3) = |(3,-5)|\,|(-4,3)| \cos\theta$

$= \sqrt{34}\cdot 5 \cdot \cos\theta$

$(3,-5)\cdot(-4,3) = -12-15 = -27$

$\therefore \cos\theta = \dfrac{-27}{5\sqrt{34}}$

#14

$|7+12i| = \sqrt{7^2+12^2} = \sqrt{49+144} = \sqrt{193}$

#17 $(2-2i) = 2(1-i)$ and $(1-i)^2 = -2i$

Thus $(2-2i)^{333} = 2^{333}(1-i)^{333}$

$= 2^{333}(1-i)^{2\cdot 166}\cdot(1-i)$

$= 2^{333}\cdot(-2i)^{166}\cdot(1-i)$

$= 2^{333}\cdot 2^{166}\cdot i^{166}\cdot(1-i)$

$= 2^{499}(i^4)^{41}\cdot i^2\,(1-i)$

$= 2^{499}\cdot(-1)\cdot(1-i)$

$= 2^{499}(-1+i)$

#19

$$-2 = 2(\cos\pi + i\sin\pi)$$

$$z^4 = 2(\cos\pi + i\sin\pi)$$

$$z = \sqrt[4]{2}\left(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}\right)$$

or $\sqrt[4]{2}\left(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}\right)$

or $\sqrt[4]{2}\left(\cos\frac{5\pi}{4} + i\sin\frac{5\pi}{4}\right)$

or $\sqrt[4]{2}\left(\cos\frac{7\pi}{4} + i\sin\frac{7\pi}{4}\right)$

#20 $4i = 4\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$

$$z^3 = 4\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$$

$$z = \sqrt[3]{4}\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$$

or $\sqrt[3]{4}\left(\cos\frac{5}{6}\pi + i\sin\frac{5}{6}\pi\right)$

or $\sqrt[3]{4}\left(\cos\frac{9}{6}\pi + i\sin\frac{9}{6}\pi\right)$

$\parallel$

$\sqrt[3]{4}\left(\cos\frac{3}{2}\pi + i\sin\frac{3}{2}\pi\right)$

#44 $z^3 = a \quad (a>0)$

$= a(\cos 0 + i\sin 0)$

$z = \sqrt[3]{a}(\cos 0 + i\sin 0)$

or $\sqrt[3]{a}\left(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}\right)$

or $\sqrt[3]{a}\left(\cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3}\right)$

$= \left\{\sqrt[3]{a},\ \sqrt[3]{a}\left(\frac{-1+\sqrt{3}i}{2}\right),\ \sqrt[3]{a}\left(\frac{-1-\sqrt{3}i}{2}\right)\right\}$